Grade 12 Mathematics
Revision Questions

(Including Solutions)

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Algebra Questions

Easy Algebra

Question A1:

Solve for x:

$$(3-x)(5-x)=3$$

Hint

Expand brackets and rearange the expression into standard form: $ax^2+bx+c=0$

Question A2:

Solve for x and y simultaneously:

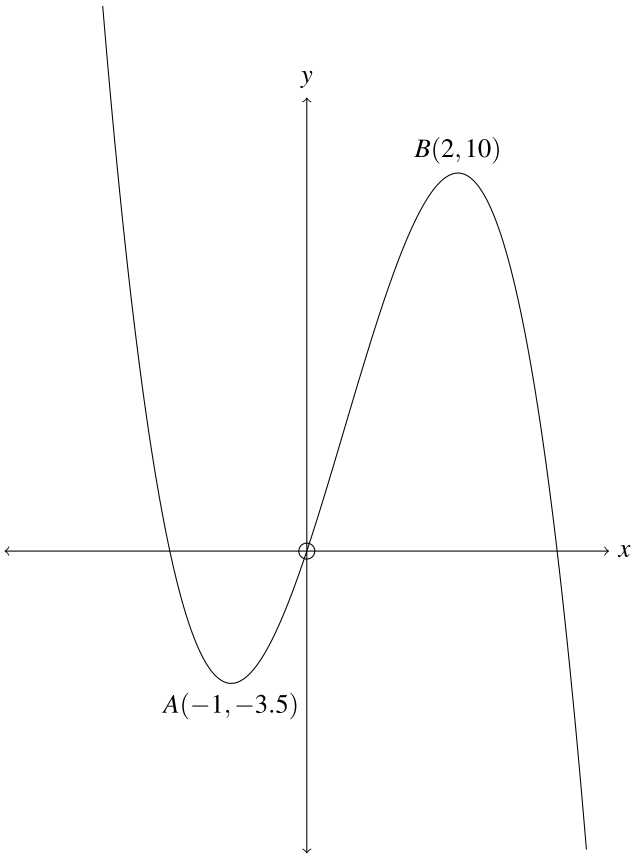
$$3y = 2x$$
$$x^2-y^2+2x-y=1$$

Hint

Let $y=\frac{2}{3}x$ then substitute y, in $x^2-y^2+2x-y=1$, with $\frac{2}{3}x$.

Question A3:

Given the graph of $f(x) = -x^3 + ax^2 + bx$, below, determine a and b.

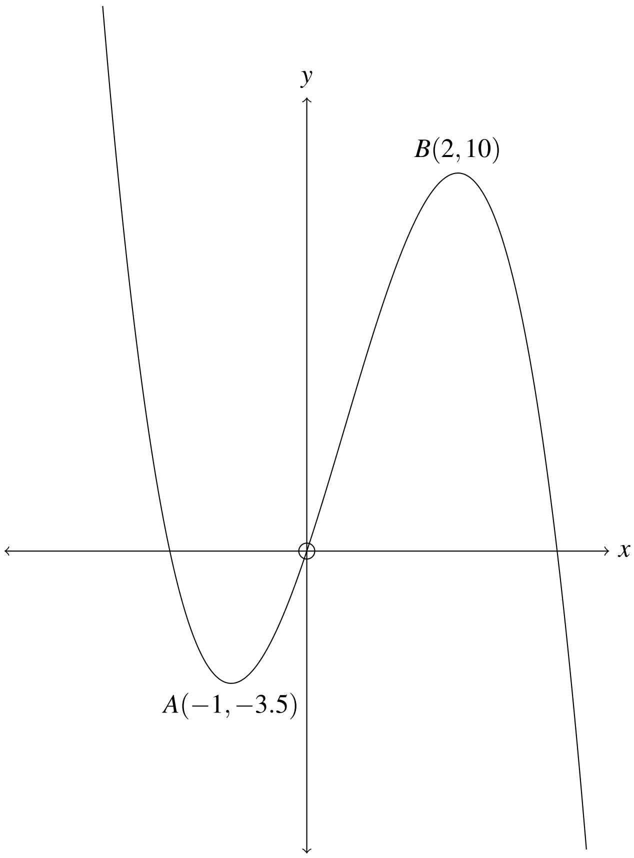


Hint

You have two unknowns and you are given two points on the graph. Solve for a and b using simultaneous equations.

Question A4:

Determine the average gradient of $f(x) = -x^3 + ax^2 + bx$, below, between the points A and B.



Hint

The average gradient of $f(x)$, between the points A and B, is equal to the gradient of the straight line that runs through A and B.

Question A5:

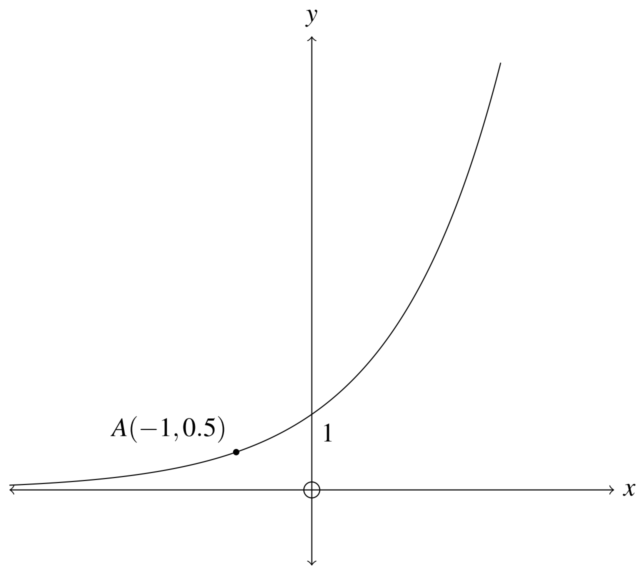
Find the gradient of $f(x) = \frac{4}{\sqrt{x}} - \frac{x^3}{9}$ at $x = 3$

Hint

Express the terms of $f(x)$ in standard form, ie. ax^n and then use the polynomial rule to differentiate each term.

Question A6:

Given the graph of $f(x) = a^x$, find a.



Hint

A negitive exponent inverts the base.

Question A7:

If $f(x) = 2^x$ and $g(x) = 100.3^x$, determine the value of x for which $f(x) = g(x)$.

Hint

Set $f(x) = g(x)$ and take logs of both sides.

Question A8:

Given the following arithmetic sequence:

$$1-p; \quad 2p-3; \quad p+5; \quad ...$$

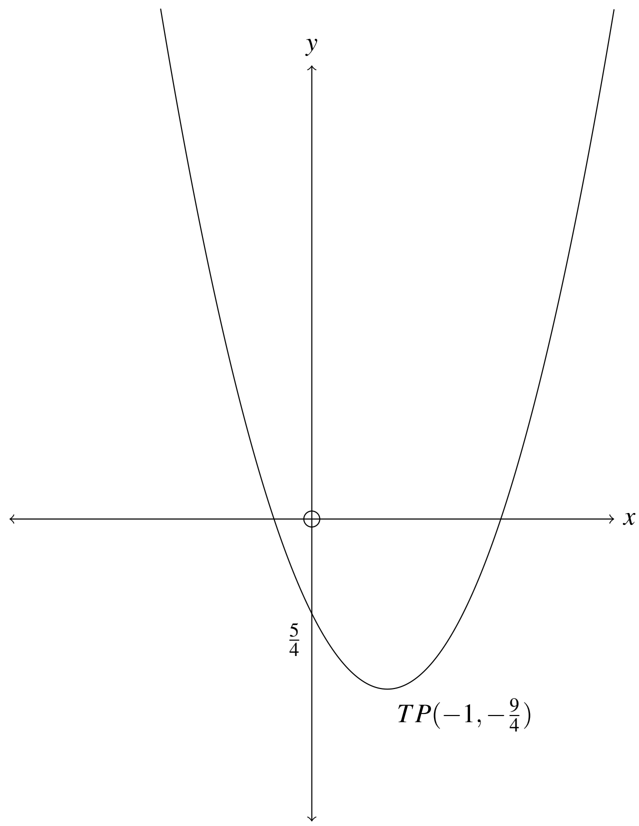
Calculate the value of p.

Hint

Think of two different ways to express the common difference in terms of p.

Question A9:

Given $f(x)$ below, determine $f\left(\frac{7}{2}\right)$:



Hint

Use the form: $f(x) = a(x-p)^2+q$ to determine the function.

Question A10:

Find the x-coordinate of the inflection point of $f(x) = -x^3+x^2+8x-12$.

Hint

The inflection point occurs when the rate of change of the function changes. In other words, find the turning point of the derivative.

Question A11:

Simplify completely:

$$\left(1+\sqrt{2x^2}\right)^2-\sqrt{8x^2}$$

Hint

$$\sqrt{8}=2\sqrt{2}$$

Medium Algebra

Question A12:

Solve for x:

$$3x+\frac{1}{x}=4$$

Hint

Once you have rearrange you trinomial into the standard form of ax^2+bx+c, you will see that $a=3$, so our brackets will have to look like this: $(3x\quad)(x\quad)$. Now all you have to do is split c up in such a way that, when you expand the brackets, you get b.

Question A13:

Solve for x:

$$15x^2+19x-8=0$$

Hint

Once you have rearrange you trinomial into the standard form of ax^2+bx+c, split a up as follows: $(5x\quad)(3x\quad)$. Now all you have to do is split c up in such a way that, when you expand the brackets, you get b. In general you would use trial and error to see which is the best way to split a up.

Question A14:

Solve for x:

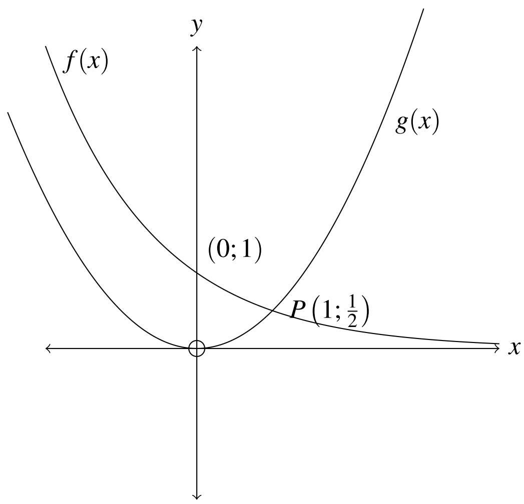
$$13x - 4 < 9x^2$$

Hint

Rearrange the expression into standard form: $0 < ax^2 + bx + c$. Find the roots and display them on a number line. Use the number line to determine which x values satisfy the expression.

Question A15:

$f(x) = a^x$ and $g(x) = bx^2$. Find a and b.

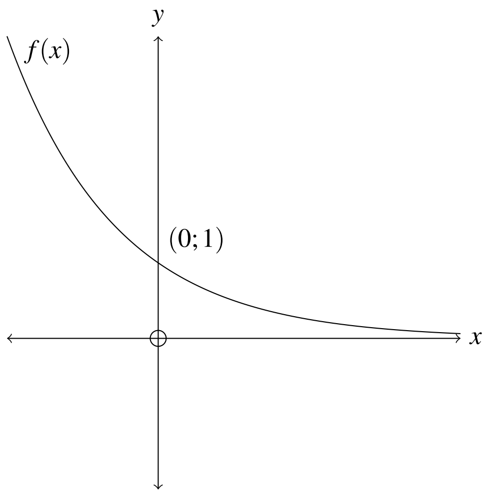


Hint

You have a point on each graph and there is only one unknown in each function.

Question A16:

Determine the x-values for which $f^{-1}(x) > 0$.

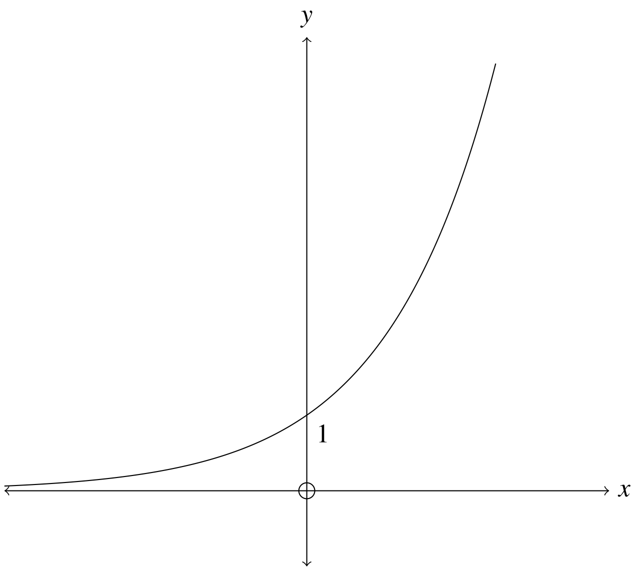


Hint

To find the inverse you need to reflect f about the line $y = x$. To help visulize what this will look like, turn your phone onto it's left side. The x-axis is now the y-axis and the y-axis is now the x-axis. However, take note that the positive and negative values on the new x-axis are swopped round (+ on the left, - on the right).

Question A17:

Given the graph of $f(x) = 2^x$, find $f^{-1}(3.5)$, where $f^{-1}(x)$ is the inverse of $f(x)$.



Hint

In general, to find the inverse of a function, swop the x's and the y's and then rearrange to make y the subject of the formula. The inverse of an exponential is a log function.

Question A18:

Find the 299th term in following sequence:

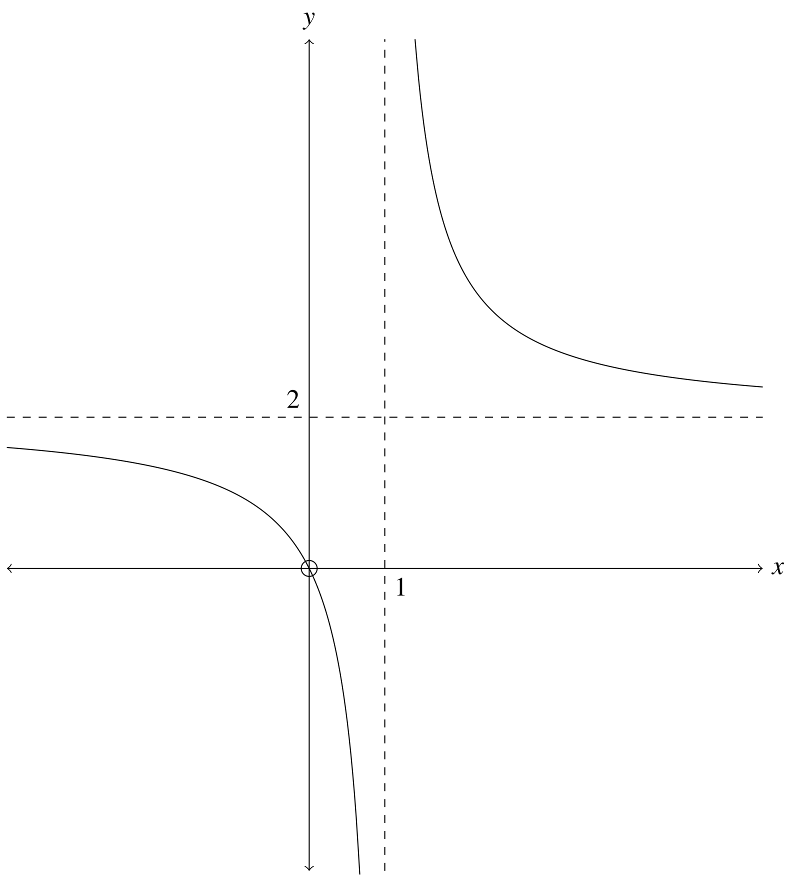
$$6;\ \ 6;\ \ 2;\ \ -6;\ \ -18;\ \ ...$$

Hint

To find T_n in the form of $an^2 + bn + c$, find the second difference of the sequence(Find the difference between each term and then find the difference between those differences). Equate it to $2a$. Now you only have to find b and c.

Question A19:

Below is a sketch of $g(x)$. If $g(x-1) = \frac{a}{x-b} + c$, determine the values of a,b and c.



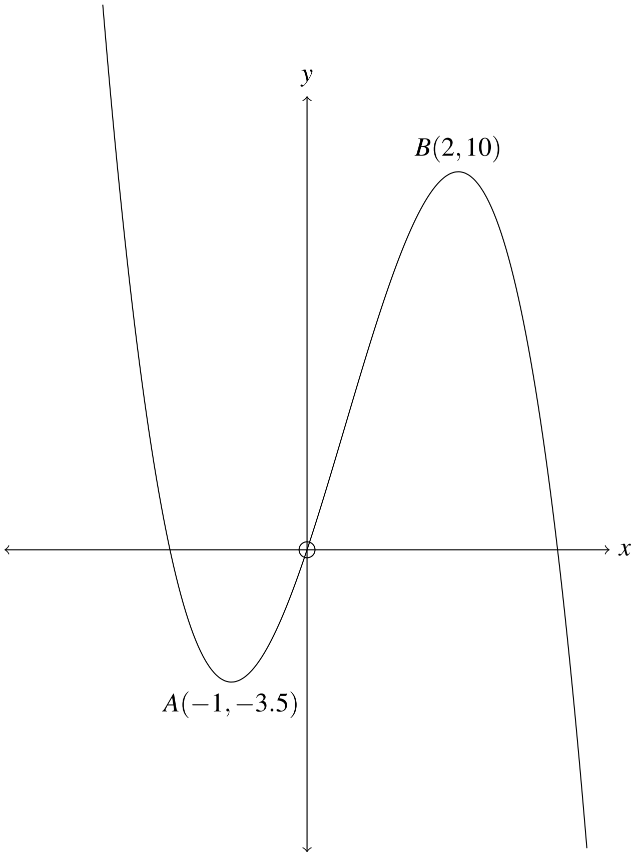
Hint

Subtracting a value from x shifts the function to the right. Adding a value to x shifts the function to the left. Subtracting a value from the whole function shifts the function downwards. Adding a value to the whole function shifts the function upwards.

You know what a standard hypebola looks like so you can determine $g(x)$ by looking at how the standard hyperbola has been shifted. You then also need to find a by substituting a point into $g(x)$. From there it is one more step to find $g(x-1)$.

Question A20:

Given the graph of $f(x) = -x^3 + \frac{3}{2}x^2 + 6x$, below, for which values of p will $g(x) = -x^3 + \frac{3}{2}x^2 + 6x + p$ have ONE real root?



Hint

For $g(x)$ to have one real root, both turning points have to be either above or below the x-axis.

Difficult Algebra

Question A21:

Determine the y coordinate of the turning point of p if $p(x) = f(3x)$ and $f(x) = -2x^2 + 8x + 10$.

Hint

Replace each x with $3x$, in $f(x)$ and then simplify.

Question A22:

Calculate the **exact** value (do not round off) of:

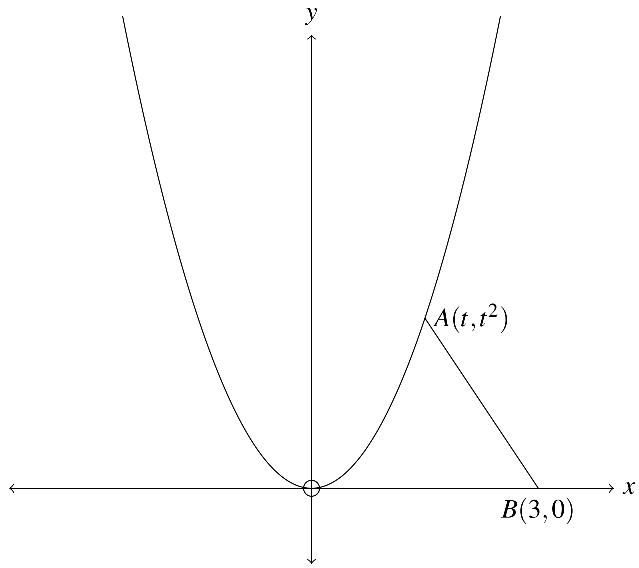
$$\frac{\sqrt{10^{2009}}}{\sqrt{10^{2011}}-\sqrt{10^{2007}}}$$

Hint

$$\sqrt{a.b}=\sqrt{a}.\sqrt{b}$$

Question A23:

Give the graph of $y=x^2$ and a line running from $A(t,t^2)$ to $B(3,0)$. Determine the value of t which minimizes the length of AB.



Hint

Use pythogorous to find an expression for AB^2 (the value of t that minimizes AB is the same value that minimizes AB^2). Use differentiation to find the value of t that minimizes AB^2.

Question A24:

Calculate the value of $1234567893 \times 1234567894 - 1234567895 \times 1234567892$. You won't be able to use a calculator for this.

Hint

Let 1234567890 = x. Rewrite the expression in terms of x.

Question A25:

Calculate the value of:

$$\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+\frac{1}{4\times 5}+...+\frac{1}{2008\times 2009}$$

Hint

Sum the first two terms of the series, then the first three and then the first four.

Algebra Solutions

Solution A1:

$$\begin{aligned}(3-x)(5-x) &= 3\\ \therefore 15-5x-3x+x^2 &= 3\\ \therefore x^2-8x+12 &= 0\\ \therefore (x-6)(x-2) &= 0\\ x=6 \qquad \text{OR} \qquad x &= 2\end{aligned}$$

Solution A2:

Part 1:

$$\begin{aligned}\text{Let} \qquad y &= \frac{2}{3}x\\ \therefore x^2-(\frac{2}{3}x)^2+2x-(\frac{2}{3}x) &= 1\\ \therefore \frac{5}{9}x^2+\frac{4}{3}x-1 &= 0\\ \therefore 5x^2+12x-9 &= 0\\ \therefore (5x-3)(x+3) &= 0\\ \therefore x=\frac{3}{5} \qquad &\text{OR} \qquad x=-3\\ \therefore y=\frac{2}{3} \qquad \text{when} \qquad x=\frac{3}{5} \qquad &\text{OR} \qquad y=-2 \qquad \text{when} \qquad x=-3\end{aligned}$$

(3/5,2/3) or (-3,-2)

Solution A3:

Substituting A into $f(x)$, we get:

$$\begin{aligned} -3.5 &= -(-1)^3 + a(-1)^2 + b(-1) \\ \therefore -3.5 &= 1 + a - b \\ \therefore -4.5 &= a - b \end{aligned}$$

Substituting B into $f(x)$, we get:

$$\begin{aligned} 10 &= -(2)^3 + a(2)^2 + b(2) \\ \therefore 10 &= -8 + 4a + 2b \\ \therefore 18 &= 4a + 2b \\ \therefore 9 &= 2a + b \\ \therefore 9 - 2a &= b \end{aligned}$$

We sunstitute $b = 9 - 2a$ back into the first equation to get:

$$\begin{aligned} -4.5 &= a - (9 - 2a) \\ \therefore -4.5 &= a - 9 + 2a \\ \therefore 4.5 &= 3a \\ \therefore a &= \frac{3}{2} \end{aligned}$$

We sunstitute $a = \frac{3}{2}$ back into $b = 9 - 2a$ to get:

$$\begin{aligned} b &= 9 - 2\left(\frac{3}{2}\right) \\ \therefore b &= 6 \end{aligned}$$

Solution A4:

$$\begin{aligned} m_{ave} &= m_{AB} \\ \therefore m_{ave} &= \frac{10 - (-3.5)}{2 - (-1)} \\ \therefore m_{ave} &= \frac{13.5}{3} \\ \therefore m_{ave} &= 4.5 \end{aligned}$$

Solution A5:

$$
\begin{aligned}
f(x) &= \frac{4}{\sqrt{x}} - \frac{x^3}{9} \\
\therefore f(x) &= 4x^{-\frac{1}{2}} - \frac{1}{9}x^3 \\
\therefore f'(x) &= -2x^{-\frac{3}{2}} - \frac{1}{3}x^2 \\
\therefore f'(3) &= -2(3)^{-\frac{3}{2}} - \frac{1}{3}(3)^2 \\
\therefore f'(3) &= -2.27^{-\frac{1}{2}} - 3 \\
\therefore f'(3) &= -2.\frac{1}{27^{\frac{1}{2}}} - 3 \\
\therefore f'(3) &= -2.\frac{1}{\sqrt{27}} - 3 \\
\therefore f'(3) &= -3.38
\end{aligned}
$$

Solution A6:

We have $a^{-1} = \frac{1}{2}$, therefore $a^1 = 2$, therefore $a = 2$.

Solution A7:

$$
\begin{aligned}
f(x) &= g(x) \\
\therefore 2^x &= 100.3^x \\
\therefore \log(2^x) &= \log(100.3^x) \\
\therefore x\log 2 &= \log 100 + \log 3^x \\
\therefore x\log 2 &= \log 100 + x\log 3 \\
\therefore x\log 2 - x\log 3 &= \log 100 \\
\therefore x(\log 2 - \log 3) &= \log 100 \\
\therefore x(\log 2/3) &= \log 100 \\
\therefore x &= \frac{\log 100}{\log 2/3} \\
\therefore x &= -11.36
\end{aligned}
$$

Solution A8:

We know that the common difference is the same between all terms, hence the name. Therefore, we can express the common difference, for this this sequence, in terms of p in two different ways and equate them:

$$
\begin{aligned}
T_2 - T_1 &= T_3 - T_2 \\
\therefore (2p-3) - (1-p) &= (p+5) - (2p-3) \\
\therefore 2p - 3 - 1 + p &= p + 5 - 2p + 3 \\
\therefore 4p &= 12 \\
\therefore p &= 3
\end{aligned}
$$

Solution A9:

Using the form $f(x) = a(x-p)^2 + q$, and the co-ordinates of the turning point, $(p,q) = (-1, -\frac{9}{4})$, we first determine a:

$$\begin{aligned} f(x) &= a(x-p)^2 + q \\ \therefore \frac{5}{4} &= a((0)-(1))^2 + \left(-\frac{9}{4}\right) \\ \therefore \frac{5}{4} &= a - \frac{9}{4} \\ \therefore 1 &= a \end{aligned}$$

We now determine the function by substituting the values for a, p and q into the form $f(x) = a(x-p)^2 + q$.

$$\begin{aligned} \therefore f(x) &= (x-1)^2 - \frac{9}{4} \\ \therefore f\left(\frac{7}{2}\right) &= \left(\left(\frac{7}{2}\right) - 1\right)^2 - \frac{9}{4} \\ \therefore f\left(\frac{7}{2}\right) &= 4 \end{aligned}$$

Solution A10:

$$\begin{aligned} f(x) &= -x^3 + x^2 + 8x - 12 \\ \therefore f'(x) &= -3x^2 + 2x + 8 \\ \therefore f''(x) &= -6x + 2 \end{aligned}$$

To find the inflection point, we set the second derivitive to 0 and solve for x.

$$\begin{aligned} 0 &= -6x + 2 \\ \therefore x &= \frac{1}{3} \end{aligned}$$

Solution A11:

$$\begin{aligned} &\left(1 + \sqrt{2x^2}\right)^2 - \sqrt{8x^2} \\ =&\left(1 + 2\sqrt{2x^2} + 2x^2\right) - \sqrt{8x^2} \\ =&1 + 2\sqrt{2x^2} + 2x^2 - 2\sqrt{2x^2} \\ =&1 + 2x^2 \end{aligned}$$

Solution A12:

$$\begin{aligned} 3x + \frac{1}{x} &= 4 \\ \therefore 3x^2 + 1 &= 4x \\ \therefore 3x^2 - 4x + 1 &= 0 \\ \therefore (3x-1)(x-1) &= 0 \\ \therefore x = \frac{1}{3} \qquad \text{OR} \qquad & x = 1 \end{aligned}$$

Solution A13:

$$\begin{aligned} 15x^2 + 19x - 8 &= 0 \\ \therefore (3x-1)(5x+8) &= 0 \\ x = \frac{1}{3} \qquad \text{OR} \qquad & x = -\frac{8}{5} \end{aligned}$$

Solution A14:

$$\begin{aligned} 13x - 4 &< 9x^2 \\ \therefore 0 &< 9x^2 - 13x + 4 \\ \therefore 0 &< (9x-4)(x-1) \end{aligned}$$

$$\therefore x < \frac{4}{9} \qquad \text{OR} \qquad x > 1$$

Solution A15:

$$\begin{aligned} b(1)^2 &= \frac{1}{2} \\ \therefore b &= \frac{1}{2} \end{aligned}$$

$$\begin{aligned} a^1 &= \frac{1}{2} \\ \therefore a &= \frac{1}{2} \end{aligned}$$

Solution A16:

$$0 < x < 1$$

Solution A17:

To find the inverse of $y = 2^x$ we swop the x's and the y's and then rearrange to make y the subject of the formula.

$$\begin{aligned} x &= 2^y \\ \therefore \log(x) &= \log(2^y) \\ \therefore \log_2(x) &= \log_2(2^y) \\ \therefore \log_2 x &= y \log_2(2) \end{aligned}$$

How do we know to take $\log_2$ of each side? Because $\log_2 2 = 1$ so we are only left with the y on the right side of the equation.

Now we have $f^{-1}(x) = \log_2 x$. Therefore:

$$\begin{aligned} f^{-1}(3.5) &= \log_2(3.5) \\ \therefore f^{-1}(3.5) &= \frac{\log(3.5)}{\log(2)} \\ \therefore f^{-1}(3.5) &= 1.81 \end{aligned}$$

Solution A18:

First we look at at the second difference. If it is constant then our general term can be written in the form $T_n = an^2 + bn + c$.

$$\begin{array}{ccccccc} 6 & & 6 & & 2 & & -6 \\ & 0 & & -4 & & -8 & \\ & & -4 & & -4 & & \end{array}$$

The second difference is equal to $2a$.Therefore:

$$\begin{aligned} 2a &= -4 \\ \therefore a &= -2 \\ \therefore T_n &= -2n^2 + bn + c \end{aligned}$$

Now we substitute two know terms into T_n to get a pair of simultaneous equations:

$$\begin{aligned} \therefore 6 &= -2(1)^2 + b(1) + c \\ \therefore 6 &= -2 + b + c \\ \therefore 8 &= b + c \end{aligned}$$

and

$$\begin{aligned} \therefore 6 &= -2(2)^2 + b(2) + c \\ \therefore 6 &= -8 + 2b + c \\ \therefore 14 &= 2b + c \end{aligned}$$

By subtracting the first equation from the second we get $b = 6$, by substituting b back into the first equation, we get $c = 2$. Therfore:

$$\begin{aligned} T_n &= -2n^2 + 6n + 2 \\ \therefore T_{299} &= -2(299)^2 + 6(299) + 2 \\ \therefore T_{299} &= -177006 \end{aligned}$$

Solution A19:

Let us first focus on finding $g(x)$. We can see that it has been shifted 1 unit to the right (subtract 1 from x) and 2 units up (add 2 to the whole function). Therefore:

$$g(x) = \frac{a}{x-1} + 2$$

To find a we have to substitute a point on the graph, into the function. (0,0) is a point on the graph. Therefore:

$$\begin{aligned} 0 &= \frac{a}{(0)-1} + 2 \\ \therefore 2 &= a \\ \therefore g(x) &= \frac{2}{x-1} + 2 \end{aligned}$$

$g(x-1)$ is simply $g(x)$ shifted 1 unit to the right. Therefore:

$$\begin{aligned} \therefore g(x-1) &= \frac{2}{(x-1)-1} + 2 \\ \therefore g(x-1) &= \frac{2}{x-2} + 2 \end{aligned}$$

$$a = 2b = 2c = 2$$

Solution A20:

For $g(x)$ to have one real root, both turning points have to be either above or below the x-axis. For B to be below the x-axis, the graph has to be shifted down by any value less than -10 (because B is 10 units above the x-axis). For A to be above the x-axis, the graph has to be shifted up by any value greater than 1 (because A is 1 unit below the x-axis). Therefore:

$$p < -10 \qquad OR \qquad p > 1$$

Solution A21:

$$\begin{aligned} p(x) &= -2(3x)^2 + 8(3x) + 10 \\ \therefore p(x) &= -18x^2 + 24x + 10 \end{aligned}$$

From here you can use The turning point formula, completing the square or differentiation to find the turning point, hence the y coordinate of the turning point.

The turning point formula:

$$
\begin{aligned}
x &= -\frac{b}{2a} \\
\therefore x &= -\frac{24}{-36} \\
\therefore x &= \frac{2}{3} \\
\therefore y &= 18 \qquad \text{by substitution}
\end{aligned}
$$

Completing the square:

$$
\begin{aligned}
p(x) &= -18x^2 + 24x + 10 \\
\therefore p(x) &= -18\left(x - \frac{2}{3}\right)^2 + 18 \\
\therefore x &= \frac{2}{3} \\
\text{and} \qquad y &= 18
\end{aligned}
$$

Differentiation:

$$
\begin{aligned}
p(x) &= -18x^2 + 24x + 10 \\
\therefore p'(x) &= -36x + 24 \\
p'(x) &= 0 \qquad \text{at the turning point} \\
\therefore 0 &= -36x + 24 \\
\therefore x &= \frac{2}{3} \\
\therefore y &= 18 \qquad \text{by substitution}
\end{aligned}
$$

Solution A22:

$$
\begin{aligned}
&\frac{\sqrt{10^{2009}}}{\sqrt{10^{2011}} - \sqrt{10^{2007}}} \\
=&\frac{\sqrt{10^{2007}}\sqrt{10^2}}{\sqrt{10^{2007}}\sqrt{10^4} - \sqrt{10^{2007}}} \\
=&\frac{\sqrt{10^{2007}}\sqrt{10^2}}{\sqrt{10^{2007}}(\sqrt{10^4} - 1)} \\
=&\frac{\sqrt{10^2}}{\sqrt{10^4} - 1} \\
=&\frac{10}{10^2 - 1} \\
=&\frac{10}{99}
\end{aligned}
$$

Solution A23:

$$\begin{aligned}AB^2 &= (t^2-0)^2+(t-3)^2\\ AB^2 &= t^4+t^2-6t+9\\ \frac{d}{dt}AB^2 &= 4t^3+2t-6\end{aligned}$$

To find the minimum value of t we set $4t^3+2t-6=0$. We also use factor theorem to find that $(t-1)$ is a factor of $4t^3+2t-6$, by substituting 1 into $4t^3+2t-6$ and finding that the expression equals 0.

$$\begin{aligned}4t^3+2t-6 &= 0\\ \therefore 2t^3+t-3 &= 0\\ \therefore (t-1)(2t^2+2t+3) &= 0\\ \therefore t &= 1 \qquad \text{There is no real solution for } 2t^2+2t+3\end{aligned}$$

Solution A24:

Let 1234567890 = x

$$\begin{aligned}&\therefore 1234567893\times1234567894-1234567895\times1234567892\\ &=(x+3)(x+4)-(x+5)(x+2)\\ &=(x^2+7x+12)-(x^2+7x+10)\\ &=x^2+7x+12-x^2-7x-10\\ &=2\end{aligned}$$

Solution A25:

We first establish a patern by summing the first two, then three and then four terms of the series:

$$\begin{aligned}\frac{1}{1\times2}+\frac{1}{2\times3}&=\frac{4}{6}\\ \frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}&=\frac{9}{12}\\ \frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\frac{1}{4\times5}&=\frac{16}{20}\end{aligned}$$

Simplifying these results, we get:

$$\begin{aligned}\text{Sum of first two terms} &= \frac{2}{3}\\ \text{Sum of first three terms} &= \frac{3}{4}\\ \text{Sum of first four terms} &= \frac{4}{5}\end{aligned}$$

We can see that the pattern is: Sum of first n terms = $\frac{n}{n+1}$. Therefore:

$$\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\frac{1}{4\times5}+\ldots+\frac{1}{2008\times2009}=\frac{2008}{2008+1}=\frac{2008}{2009}$$

Trigonometry Questions

Easy Trigonometry

Question T1:

Simplify the following:

$$\frac{\sin(90^\circ + x).\cos(180 - x).\tan(-x)}{\cos(270^\circ - x)}$$

Hint

When an angle is given in reference to the y-axis, ie. $(90+x)$, $(270+x)$, etc, it can always be reduced to $(90-x)$. You must also make the appropriate sign change for the quadrant you are in.

Question T2:

Given $f(x) = 2\sin x$, find the minimum positive value (in degrees) of θ such that $f(x+\theta) = 2\cos x$

Hint

$f(x)$ must be shifted to the left.

Question T3:

Express the following expression in terms of $\tan x$ only:

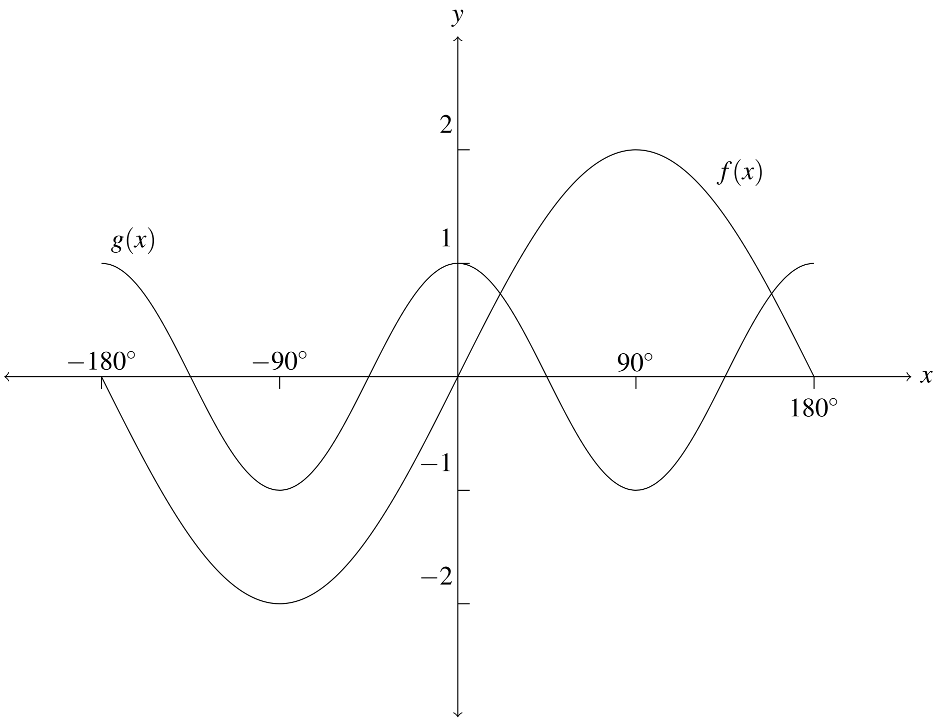
$$\frac{\sin x \cos x}{1 - \sin^2 x + \cos^2 x}$$

Hint

Use $\sin^2 x + \cos^2 x = 1$ to simplify the expression.

Question T4:

Given the following graph, where $f(x) = 2\sin x$ and $g(x) = \cos 2x$, $x \in [-180°; 180°]$:



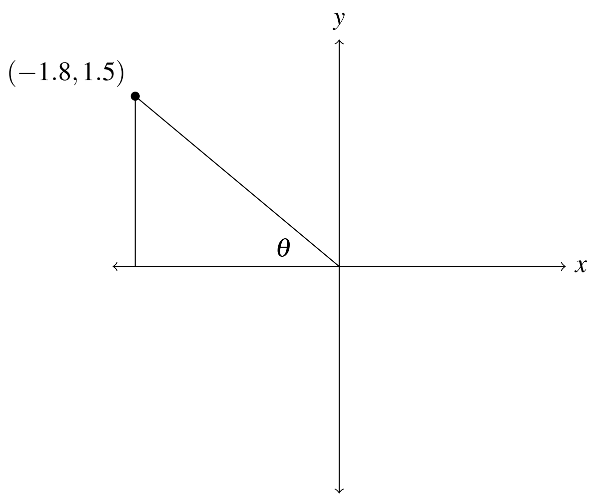
For which value of x will $f(x) - g(x) = 3$?

Hint

Where about on the graph is the distance between $f(x)$ and $g(x)$, equal to three? No working is required, the answer can be read straight off the graph.

Question T5:

Find theta.

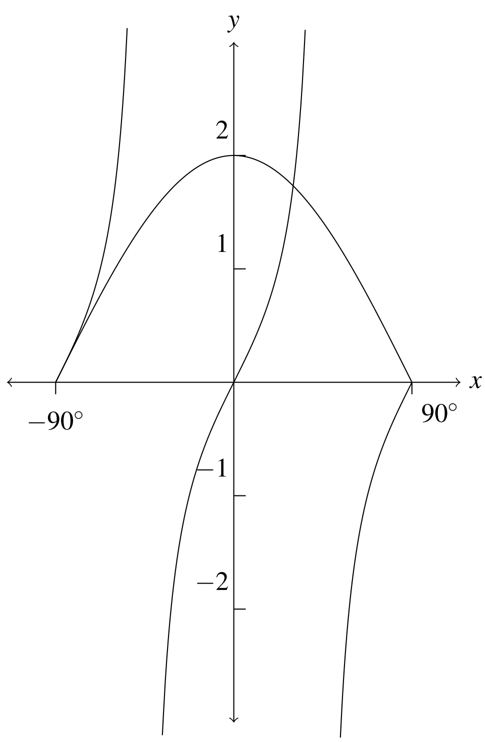


Hint

The length of the opposite and adjacent sides are given by the point (-1.8,1.5).

Question T6:

Given the following graph, where $f(x) = 2\cos x$ and $g(x) = \tan 2x$, $x \in [-90°; 90°]$:



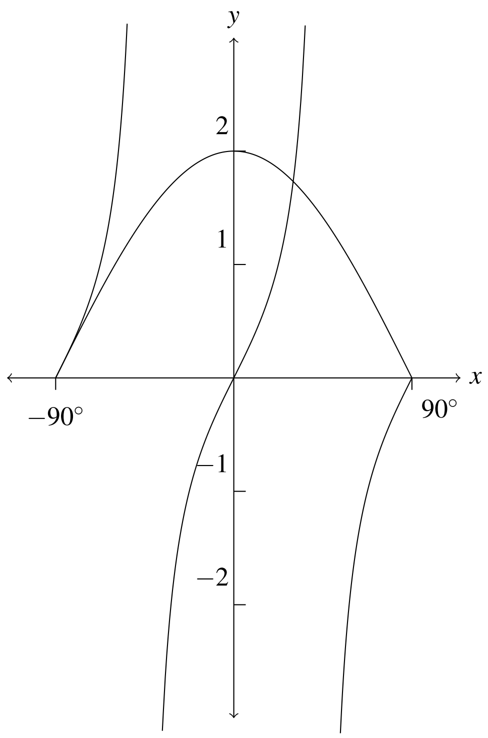
Determine the period of $f\left(\frac{x}{2}\right)$.

Hint

The period is the number of degrees that it takes for a trig function to go through one full cycle. We know this to be 360° for $\cos x$. For $\cos\left(\frac{x}{2}\right)$ the frequency has been halved. This means that the period has been doubled (the period is the inverse of the frequency).

Question T7:

Given the following graph, where $f(x) = 2\cos x$ and $g(x) = \tan 2x$, $x \in [-90°; 90°]$:



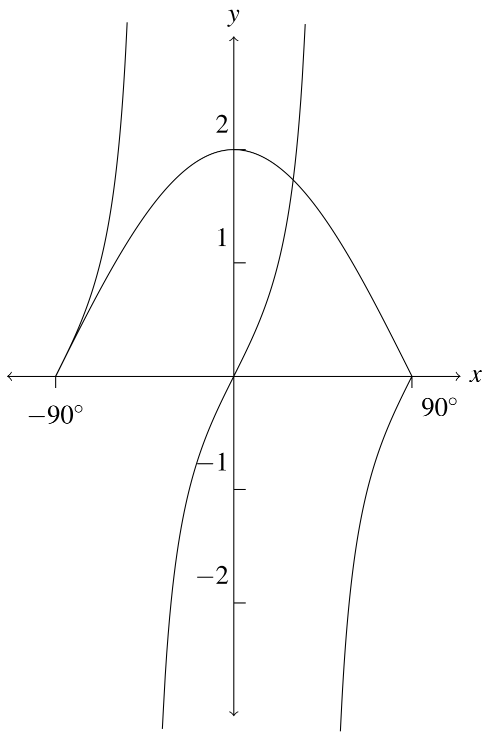
Determine the equation of the asymptote of $g(x - 25°)$, where $x \in [0°; 90°]$

Hint

First you will need to determine the equation of the asymptote of $g(x)$ where $x \in [0°; 90°]$. Then you will have to shift it either left or right.

Question T8:

Given the following graph, where $f(x) = 2\cos x$ and $g(x) = \tan 2x$, $x \in [-90°; 90°]$:



For which values of x will $2\cos x.\tan 2x > 0$?

Hint

This occurs over the intervals of x for which the signs of $f(x)$ and $g(x)$ are equal. The key is to identify the asymptotes of $g(x)$.

Question T9:

Simplify completely:

$$\sin(90° - x)\cos(180° - x) + \tan x.\cos(-x)\sin(180° + x)$$

Hint

$$\sin(x) = \cos(90° - x)$$

$$\sin^2 x + \cos^2 x = 1$$

Question T10:

If $13 \sin \theta - 5 = 0$, find $\cos \theta$

Hint

Draw a right angle triangle with angle θ. You have the lengths of the opposite and hypotenuse, now find the adjacent side.

Question T11

Solve for $x \in [0°; 360°]$ if $\frac{1}{2} \cos x = 0,435$.

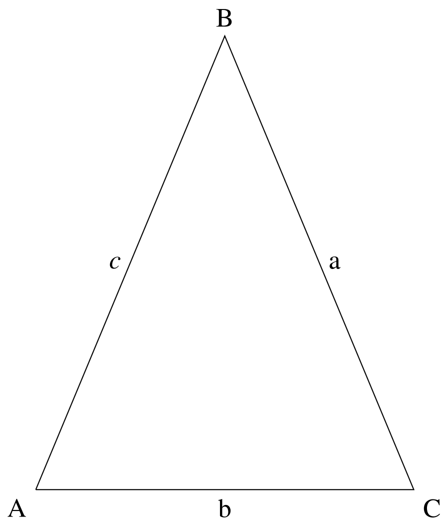
Hint

There are two values for x on the interval $[0°; 360°]$ for which this equality holds.

Medium Trigonometry

Question T12:

Triangle ABC is isosceles with AB = BC.



Express $\cos B$ in terms of a and b. Simplify as far as possible.

Hint

Cosine rule.

Question T13:

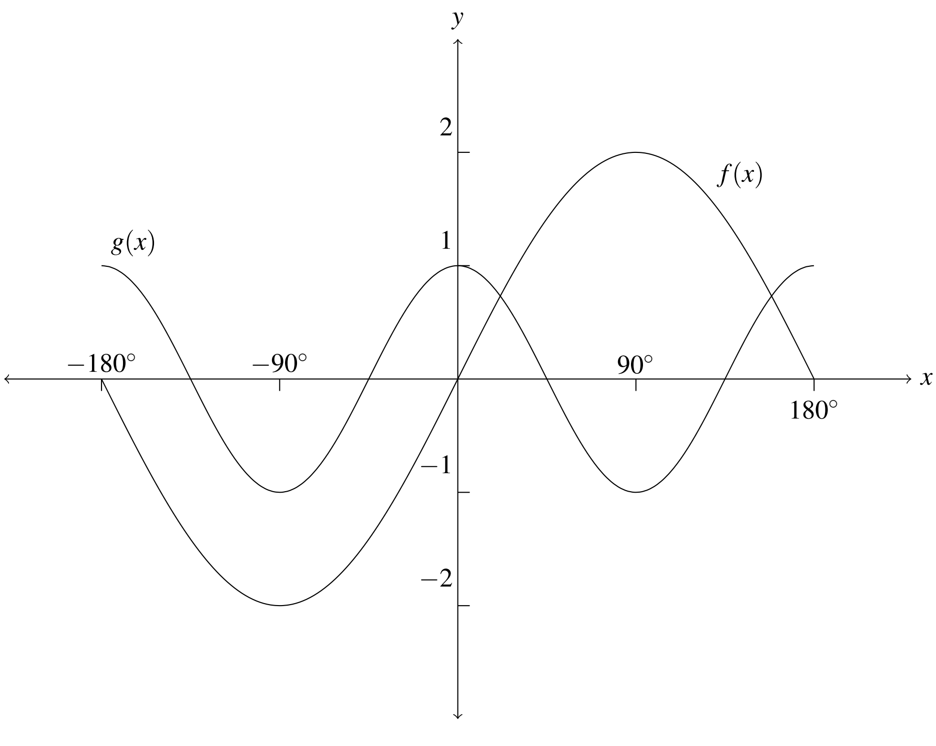
If $\cos x = \sqrt{t}$, express $\tan x$ in terms of t.

Hint

If $\cos x = \sqrt{t}$ then we can construct a right angle triangle that has an angle x, an adjacent side of length $\sqrt{t}$ and a hypotenuse of length 1. Then use the Theorem of Pythagoras to find the opposite side.

Question T14:

Given the following graph, where $f(x) = 2\sin x$ and $g(x) = \cos 2x$, $x \in [-180°; 180°]$:



For which four values of x will $g(x) - f(x) = 1$?

Hint

Where about on the graph is the distance between $f(x)$ and $g(x)$, equal to one? No working is required, the answer can be read straight off the graph.

Question T15:

Simplify the following expression to one trig ratio of θ:

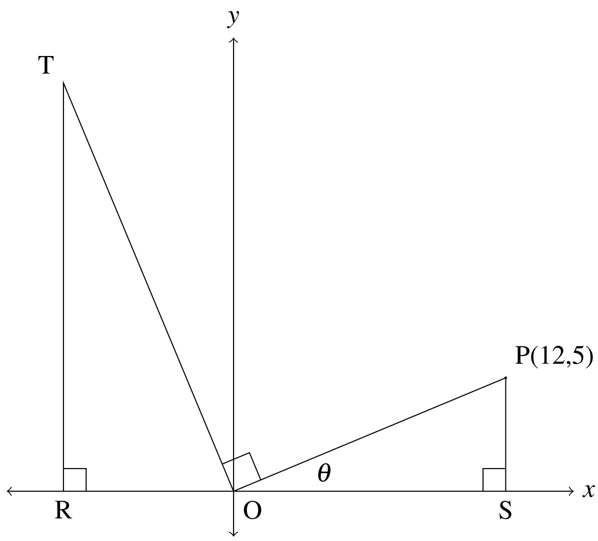
$$\frac{\sin(-\theta).\sin(180° - \theta) + \cos(90° + \theta)}{-\sin(360° - \theta) - \tan 315°}$$

Hint

If an angle is given with reference to the x-axis, eg: $\sin(180^\circ + \theta)$, then it is easy to reduce. All you do is change the angle to the acute angle and make the appropriate sign change for the quadrant it is in, eg: we first change $\sin(180^\circ + \theta)$ to $\sin(\theta)$, but it is also in the third quadrant so it gets a sign change and becomes $-\sin(\theta)$. If, however, an angle is given with reference to the y-axis, we go through the exact same two steps and then we add a third step, which is to change the trig function to its co-ratio. Eg: We change $\sin(90^\circ + \theta)$ to $\sin(\theta)$, it is in the second quadrant, so there is no sign change, and for the third step we change the the sin to a cos so that it becomes $\cos(\theta)$.

Question T16:

Find the length of OT if OR = 7.5cm.



Hint

$\angle RTO = \angle POS$

Question T17:

If $\sin 23^\circ = p$, write the following in terms of p:

$$\sin 46^\circ$$

Hint

Use $\sin 2\theta = 2\sin\theta\cos\theta$ then $\sin^2\theta + \cos^2\theta = 1$.

Question T18:

Determine the minimum and maximum values of the following:

$$f(x) = \frac{1}{3\sin^2 x + 4\cos^2 x}$$

Hint

$$\sin^2 x + \cos^2 x = 1$$

Difficult Trigonometry

Question T19:

Determine the value of:

$$\tan 1^\circ \times \tan 2^\circ \times \tan 3^\circ \times \tan 4^\circ \times ... \times \tan 87^\circ \times \tan 88^\circ \times \tan 89^\circ$$

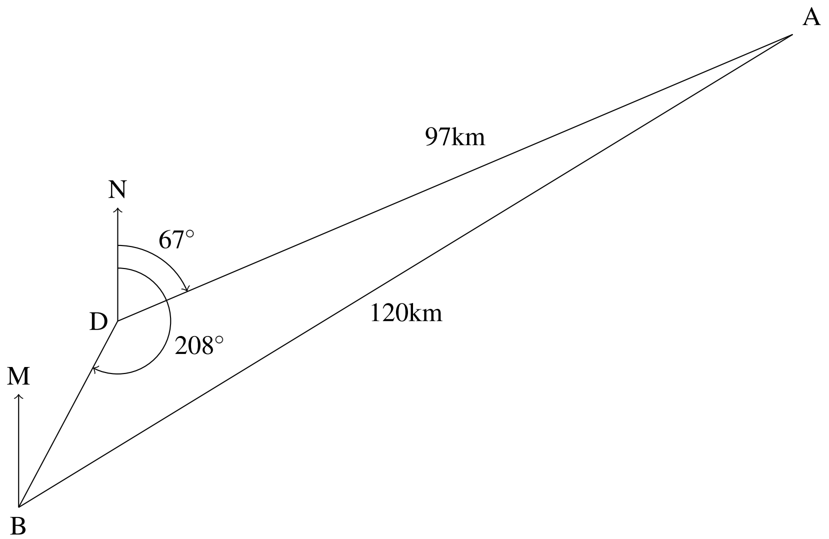
Hint

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \text{and} \qquad \sin\theta = \cos(90^\circ - \theta)$$

Question T20:

Two ships, A and B, are 120km apart. Ship A is at a bearing of 67° from D and 97km away from D. DN points due north. Ship B is at a bearing of 208° from D.

Determine the bearing of Ship A from Ship B, that is $M\hat{B}A$.



Hint

You need to find $M\hat{B}D$ and $D\hat{B}A$ separatly and then add them together. $M\hat{B}D$ can be determined without doing any working.

Question T21:

Determine the lowest positive value solution of:

$$\sin x + 2\cos^2 x = 1$$

Hint

Rewite the equation in terms of $\sin x$ only and then solve as a quadratic.

Question T22:

If $\sqrt{2} = a$ and $\sqrt{3} = b$, express $\sin 15^\circ$ in terms of a and b.

Hint

Think of two special angles such that, when one is subtracted from the other, you get 15°. Then, using the double angle formula, $\sin(x-y) = \sin x\cos y - \cos x\sin y$, express $\sin 15^\circ$ in terms of trig functions of the special angles that you have chosen.

Question T23:

If $\cos x = \sqrt{t}$, express $\sin 2x$ in terms of t.

Hint

If $\cos x = \sqrt{t}$ then we can construct a right angle triangle that has an angle x, an adjacent side of length $\sqrt{t}$ and a hypotenuse of length 1. Then use the Theorem of Pythagoras to find the opposite side. Watch out though, this triangle is for the angle x, and we are looking for $\sin 2x$. You will have to use a trig identity to express $\sin 2x$ in terms of trig functions of just x.

Question T24:

Solve the following equation for $x \in (-180°; 180°]$:

$$\frac{\sin x \cos x}{1 + \cos^2 x - \sin^2 x} = 0$$

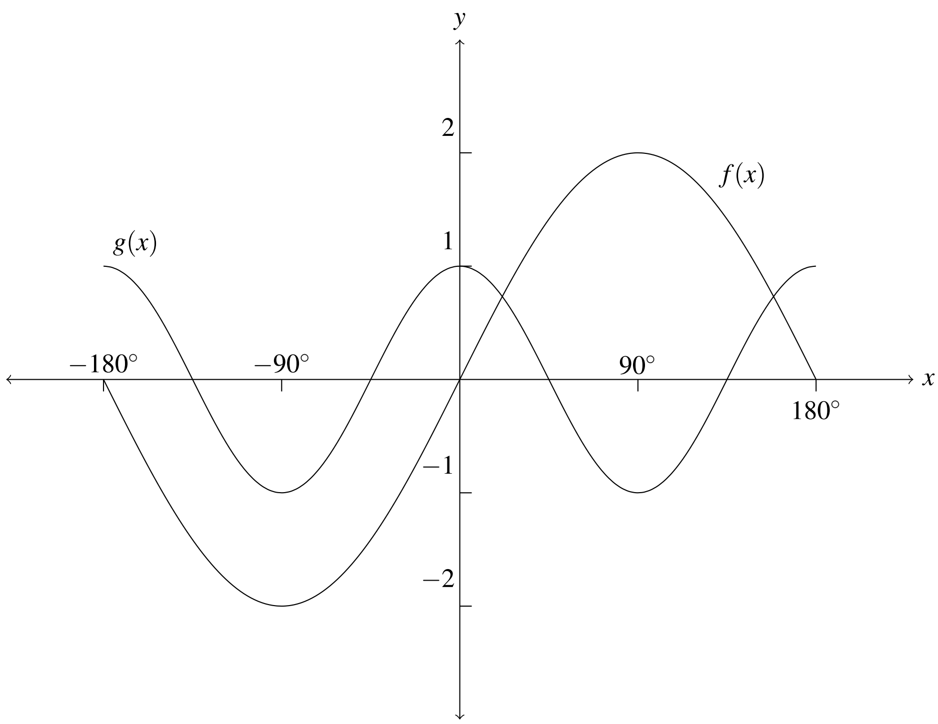
Hint

Use $\sin^2 x + \cos^2 x = 1$ to simplify the equation. Find the general solution and use it to find the values of $x \in (-180°; 180°]$ that satisfy the equation.

For $x \in (-180°; 180°]$, the round bracket means we **don't** include $-180°$ and the square bracket means we **do** include $180°$.

Question T25:

Given the following graph, where $f(x) = 2\sin x$ and $g(x) = \cos 2x$, $x \in [-180°; 180°]$:



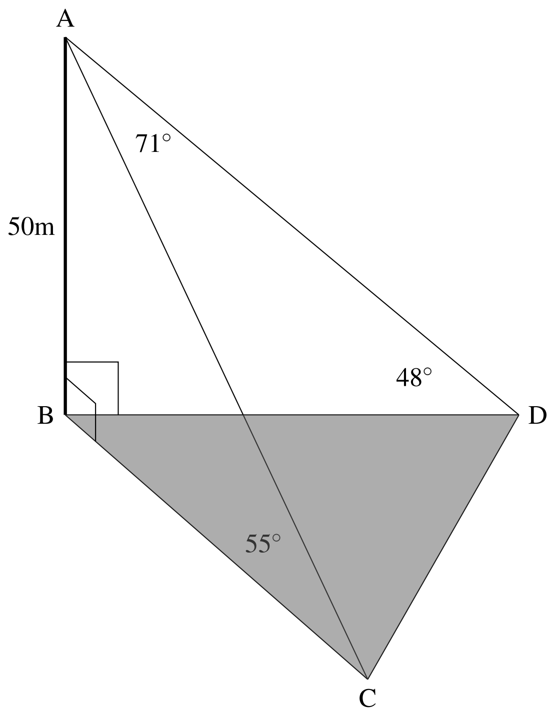
For which values of x is $f(x) > g(x)$?

Hint

$f(x)$ is greater than $g(x)$ on an interval which is defined by the points of intersection of the two functions.

Question T26:

AB is a vertical pole, 50m high, with two cables running from the top, down to two securing points on the ground, at C and D. The shaded region, $\triangle BCD$, is the ground (horizontal plane). $A\hat{C}B = 55^\circ$, $A\hat{D}B = 48^\circ$ and $C\hat{A}D = 71^\circ$.



Find the distance between the two securing points, C and D.

Hint

Visualisation is key. You have to view this diagram as 3D, not 2D. All the triangles you can see, are in different planes. Therefore, none of the angles can be related to each other. Your first step is to find the lines AC and AD. AC is the hypotenuse of the vertical triangle, $\triangle ABC$, and AD is the hypotenuse of the vertical triangle, $\triangle ABD$.

Trigonometry Solutions

Solution T1:

$$
\begin{aligned}
&\frac{\sin(90^\circ + x).\cos(180 - x).\tan(-x)}{\cos(270^\circ - x)} \\
=&\frac{\sin(90^\circ - x).(-\cos x).(-\tan x)}{-\cos(90^\circ - x)} \\
=&\frac{\cos x.(-\cos x).(-\frac{\sin x}{\cos x})}{-\sin x} \\
=&-\cos x
\end{aligned}
$$

Solution T2:

$f(x) = 2\sin x$ needs to be shifted 90° to the left for it to equal $2\cos x$. For any function, if we want to shift it to the left then we have to add a positive value to x. Therefore:

$$2\sin(x + 90^\circ) = 2\cos x$$

$$90^\circ$$

Solution T3:

$$
\begin{aligned}
&\frac{\sin x \cos x}{1 - \sin^2 x + \cos^2 x} \\
=&\frac{\sin x \cos x}{\cos^2 x + \cos^2 x} \\
=&\frac{\sin x \cos x}{2\cos^2 x} \\
=&\frac{\sin x}{2\cos x} \\
=&\frac{1}{2}\frac{\sin x}{\cos x} \\
=&\frac{1}{2}\tan x
\end{aligned}
$$

Solution T4:

To get $f(x) - g(x) = 3$, $f(x)$ has to be greater than $g(x)$ (obviously, else our result would be negative). Also, we can see that the maximum value for $f(x)$ is 2, and the minimum value of $g(x)$ is -1. So, $f(x) - g(x)$ will only equal 3 where both $f(x)$ is at it's maximum and $g(x)$ is at it's minimum. It is clear from the graph that this only happens at $x = 90^\circ$.

Solution T5:

The length of the opposite side is given by the x-coordinate of (-1.8,1.5) and the length of the adjacent side is given by the y-coordinate of (-1.8,1.5).

Therefore we have:

Length of opposite side: 1.8
Length of adjacent side: 1.5

So we have:

$$\begin{aligned} \tan\theta &= \frac{1.8}{1.5} \\ \therefore \theta &= \tan^{-1}\frac{1.8}{1.5} \\ \therefore \theta &= 50.19^\circ \end{aligned}$$

Solution T6:

For $\cos\left(\frac{x}{2}\right)$ the frequency has been halved. This means that the period has been doubled (the period is the inverse of the frequency). We know that the period for $\cos x$ is 360°. Therefore the period for $\cos\left(\frac{x}{2}\right)$ must be 720°

Solution T7:

First you will need to determine the equation of the asymptote of $g(x)$ where $x \in [0^\circ; 90^\circ]$. It is $x = 45^\circ$. Then, if we subtract 25° from x in our function: $g(x-25^\circ)$, we will shift $g(x)$ to the right by 25°. Therefore the asymptote will also shift 25° to the right, giving us a new equation of $x = 70^\circ$

Solution T8:

$f(x)$ is positive for the whole interval of $[-90^\circ; 90^\circ]$, so we only have to determine the values of x for which $g(x)$ is positive. To do this have to identify the asymtotes of $g(x)$. Because the frequency of $g(x)$ has been doubled, the period has been halved, therefore they occur at -45° and 45°. Therefore $g(x)$ is positive over the following intervals:

$$\begin{aligned} -90^\circ < &x < -45^\circ \\ 0^\circ < &x < 45^\circ \end{aligned}$$

Which means that $2\cos x.\tan 2x > 0$ over those same intervals.

Solution T9:

First we note that, if $\sin(x) = \cos(90° - x)$, then we have:

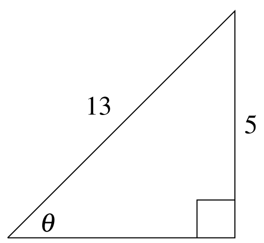
$$\begin{aligned} \sin(90° - x) &= \cos(90° - (90° - x)) \\ \therefore \sin(90° - x) &= \cos(x) \end{aligned}$$

Now we simplify:

$$\begin{aligned} &\sin(90° - x)\cos(180° - x) + \tan x.\cos(-x)\sin(180° + x) \\ =&\cos(x). - \cos(x) + \frac{\sin x}{\cos x}.\cos(x). - \sin(x) \\ =& - \cos^2(x) + \sin x. - \sin(x) \\ =& - \cos^2(x) - \sin^2(x) \\ =& - (\cos^2(x) + \sin^2(x)) \\ =& - 1 \end{aligned}$$

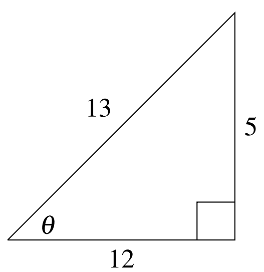
Solution T10:

Draw a right angle triangle with angle θ. You have the lengths of the opposite and hypotenuse. Fill them in and use the theorem of Pythagoras to determine that the adjacent side is 12.



$$\begin{aligned} \text{adjacent}^2 &= 13^2 - 5^2 \\ \therefore\ \text{adjacent} &= \sqrt{13^2 - 5^2} \\ \therefore\ \text{adjacent} &= 12 \end{aligned}$$

Therefore we have:



Therefore, $\cos\theta = \frac{12}{13}$.

Solution T11

$$
\begin{aligned}
\frac{1}{2}\cos x &= 0,435 \\
\therefore \cos x &= 0,87 \\
\therefore x &= \cos^{-1} 0,87 \\
\therefore x &= 29.54
\end{aligned}
$$

By making a rough sketch of $\cos x$, it can be seen that the equality will also hold for $360° - 29.54° = 330.46°$.

Solution T12:

$$
\begin{aligned}
b^2 &= a^2 + c^2 - 2ac\cos B \\
& \text{ABC is isosceles, therefore } a = c \\
\therefore b^2 &= a^2 + a^2 - 2a^2\cos B \\
\therefore \frac{b^2 - 2a^2}{-2a^2} &= \cos B \\
\therefore \cos B &= 1 - \frac{b^2}{2a^2}
\end{aligned}
$$

Solution T13:

To find the opposite side of the right angle triangle, we use the Theorem of Pythagoras:

$$
\begin{aligned}
(\text{opposite side})^2 &= (1)^2 - (\sqrt{t})^2 \\
\therefore \text{opposite side} &= \sqrt{(1)^2 - (\sqrt{t})^2} \\
&= \sqrt{1-t}
\end{aligned}
$$

We know that $\tan x$ is equal to opposite over adjacent, therefore:

$$
\begin{aligned}
\tan x &= \frac{\sqrt{1-t}}{\sqrt{t}} \\
&= \sqrt{\frac{1-t}{t}}
\end{aligned}
$$

Solution T14:

For $x = -180°, 0°$ and $180°$, $g(x)$ has a value of 1 and $f(x)$ has a value of 0. Therefore, at these three values, $g(x) - f(x)$ is equal to 1. At $x = -90°$, we can see that $g(x) = -1$ and $f(x) = -2$. Therefore $g(x) - f(x)$ is also equal to 1 at $x = -90°$.

Solution T15:

$$\begin{aligned}
&\frac{\sin(-x).\sin(180^\circ-x)+\cos(90^\circ+x)}{-\sin(360^\circ-x)-\tan 315^\circ}\\
=&\frac{-\sin x.\sin x-\sin x}{-(-\sin x)-(-\tan 45^\circ)}\\
=&\frac{-\sin x.\sin x-\sin x}{\sin x+1}\\
=&\frac{-\sin x(\sin x+1)}{\sin x+1}\\
=&-\sin x
\end{aligned}$$

Solution T16:

OP = 13 (By Pythagoras)

$\angle TOR$ = $90^\circ-\theta$ (Angle sum of straight line)

Therefore $\angle RTO$ = θ (Angle sum of triangle)

So we have:

$$\begin{aligned}
\frac{OT}{OR}&=\frac{13}{5}\\
\therefore OT&=OR\frac{13}{5}\\
\therefore OT&=7.5\frac{13}{5}\\
\therefore OT&=19.5
\end{aligned}$$

Solution T17:

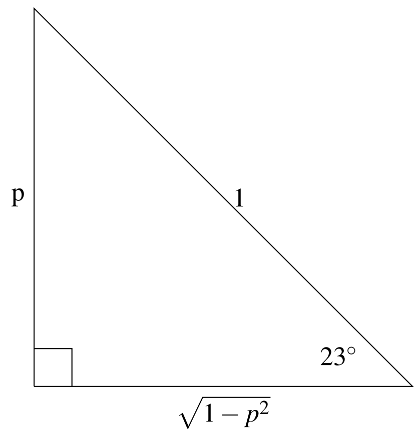
First we notice that 46° is double 23° so the double angle formula, $\sin 2\theta=2\sin\theta\cos\theta$, comes to mind. Using this formula we get:

$$\sin 46^\circ=2\sin 23^\circ\cos 23^\circ$$

Now we need to express $\cos 23^\circ$ in terms of p. To do this we use $\sin^2\theta+\cos^2\theta=1$:

$$\begin{aligned}
\sin^2\theta+\cos^2\theta&=1\\
\therefore \sin^2 23^\circ+\cos^2 23^\circ&=1\\
\therefore p^2+\cos^2 23^\circ&=1\\
\therefore \cos^2 23^\circ&=1-p^2\\
\therefore \cos 23^\circ&=\sqrt{1-p^2}
\end{aligned}$$

NOTE: Pythagoras could have been used just as effectively to arrive at $\cos 23^\circ=\sqrt{1-p^2}$:



Putting this all together, we get:

$$\sin 46^\circ = 2\sin 23^\circ \cos 23^\circ$$
$$\therefore \sin 46^\circ = 2p\sqrt{1-p^2}$$

Solution T18:

Rewrite the expression in terms of either $\sin x$ or $\cos x$ (we will consider $\sin x$), using the trig identity $\sin^2 x + \cos^2 x = 1$:

$$f(x) = \frac{1}{3\sin^2 x + 4\cos^2 x}$$
$$\therefore f(x) = \frac{1}{3\sin^2 x + 4(1-\sin^2 x)}$$
$$\therefore f(x) = \frac{1}{3\sin^2 x + 4 - 4\sin^2 x}$$
$$\therefore f(x) = \frac{1}{4-\sin^2 x}$$

We know that the maximum value of $\sin x$ is 1, hence the maximum value of $\sin^2 x$ is 1. We also know that the minimum value of $\sin^2 x$ is 0.

$f(x)$ will be at it's maximum when its denominator is at its minimum. This occurs when $\sin^2 x$ is at it's maximum:

$$f(x) = \frac{1}{4-1}$$
$$\therefore f(x) = \frac{1}{3}$$

$f(x)$ will be at it's minimum when its denominator is at its maximum. This occurs when $\sin^2 x$ is at it's minimum:

$$f(x) = \frac{1}{4-0}$$
$$\therefore f(x) = \frac{1}{4}$$

Solution T19:

$$
\begin{aligned}
&\tan 1^\circ \times \tan 2^\circ \times \tan 3^\circ \times \tan 4^\circ \times ... \times \tan 87^\circ \times \tan 88^\circ \times \tan 89^\circ \\
=&\left(\frac{\sin 1^\circ}{\cos 1^\circ}\right)\left(\frac{\sin 2^\circ}{\cos 2^\circ}\right)...\left(\frac{\sin 45^\circ}{\cos 45^\circ}\right)...\left(\frac{\sin 88^\circ}{\cos 88^\circ}\right)\left(\frac{\sin 89^\circ}{\cos 89^\circ}\right) \\
=&\left(\frac{\sin 1^\circ}{\cos 1^\circ}\right)\left(\frac{\sin 2^\circ}{\cos 2^\circ}\right)...\left(\frac{\sin 45^\circ}{\cos 45^\circ}\right)...\left(\frac{\sin(90^\circ - 2^\circ)}{\cos(90^\circ - 2^\circ)}\right)\left(\frac{\sin(90^\circ - 1^\circ)}{\cos(90^\circ - 1^\circ)}\right) \\
=&\left(\frac{\sin 1^\circ}{\cos 1^\circ}\right)\left(\frac{\sin 2^\circ}{\cos 2^\circ}\right)...\left(\frac{\sin 45^\circ}{\cos 45^\circ}\right)...\left(\frac{\cos 2^\circ}{\sin 2^\circ}\right)\left(\frac{\cos 1^\circ}{\sin 1^\circ}\right) \\
=&\tan 45^\circ \\
=&1
\end{aligned}
$$

Solution T20:

We can see that $N\hat{D}B$ is equal to 152° $(360^\circ - 208^\circ)$. We know that the sum of interior angles equal 180°, therefore $M\hat{B}D = 28^\circ$

To determine $D\hat{B}A$, we use the sin rule:

$$
\begin{aligned}
\frac{\sin A}{a} &= \frac{\sin B}{b} \\
\therefore \frac{\sin D\hat{B}A}{97} &= \frac{\sin 141^\circ}{120} \\
\therefore D\hat{B}A &= \sin^{-1}\left(97\frac{\sin 141^\circ}{120}\right) \\
\therefore D\hat{B}A &= 30.58^\circ
\end{aligned}
$$

Therefore $M\hat{B}D$ = 28° + 30.58° = 58.58°.

Solution T21:

Using the identity $\sin^2 x + \cos^2 x = 1$, rewite the equation in terms of $\sin x$ only:

$$
\begin{aligned}
\sin x + 2\cos^2 x &= 1 \\
\therefore \sin x + 2(1 - \sin^2 x) &= 1 \\
\therefore 0 &= 2\sin^2 x - \sin x - 1
\end{aligned}
$$

If we let $\sin x = k$, we get:

$$
\begin{aligned}
2k^2 - k - 1 &= 0 \\
\therefore (2k+1)(k-1) &= 0 \\
\therefore k = -\frac{1}{2} &\qquad \text{OR} \qquad k = 1 \\
\therefore \sin x = -\frac{1}{2} &\qquad \text{OR} \qquad \sin x = 1
\end{aligned}
$$

We now have to consider the general solution for $\sin x = -\frac{1}{2}$ and $\sin x = 1$.

For $\sin x = -\frac{1}{2}$, we have:

$$
\begin{aligned}
x = \sin^{-1}\left(-\frac{1}{2}\right) + n.360^\circ \qquad & \text{OR} \qquad \left(180^\circ - \sin^{-1}\left(-\frac{1}{2}\right)\right) + n.360^\circ \qquad n \in \mathbb{Z} \\
\therefore x = -30^\circ + n.360^\circ \qquad & \text{OR} \qquad 210^\circ + n.360^\circ
\end{aligned}
$$

For $\sin x = 1$, we have:

$$
\begin{aligned}
x = \sin^{-1}(1) + n.360^\circ \qquad & \text{OR} \qquad \left(180^\circ - \sin^{-1}(1)\right) + n.360^\circ \qquad n \in \mathbb{Z} \\
\therefore x = 90^\circ + n.360^\circ \qquad & \text{OR} \qquad 90^\circ + n.360^\circ \\
\therefore x = 90^\circ + n.360^\circ &
\end{aligned}
$$

So x could be any value that satisfies any one of the these three general solutions, where n could be any interger:

$$
\begin{aligned}
x &= -30^\circ + n.360^\circ \\
x &= 210^\circ + n.360^\circ \\
x &= 90^\circ + n.360^\circ
\end{aligned}
$$

We are interested in finding the lowest positive value of x. By inspection we can see that we get the lowest positive value when we take $n = 0$ for $x = 90^\circ + n.360^\circ$. Therefore 90° is the lowest positive solution for our equation.

Solution T22:

$$
\begin{aligned}
\sin 15^\circ &= \sin(45^\circ - 30^\circ) \\
&= \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ \\
&= \frac{1}{\sqrt{2}}.\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}.\frac{1}{2} \\
&= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} \\
&= \frac{\sqrt{3} - 1}{2\sqrt{2}} \\
&= \frac{b - 1}{2a}
\end{aligned}
$$

Solution T23:

To find the opposite side of the right angle triangle, we use the Theorem of Pythagoras:

$$
\begin{aligned}
(\text{opposite side})^2 &= (1)^2 - (\sqrt{t})^2 \\
\therefore \text{opposite side} &= \sqrt{(1)^2 - (\sqrt{t})^2} \\
&= \sqrt{1 - t}
\end{aligned}
$$

Now we use a trig identity to express $\sin 2x$ in terms of trig functions of just x:

$$\sin 2x = 2 \sin x \cos x$$

We have $\cos x = \sqrt{t}$. We know that $\sin x$ is equal to opposite over hypotenuse, therefore:

$$\begin{aligned}
\sin x &= \frac{\sqrt{1-t}}{1} \\
&= \sqrt{1-t}
\end{aligned}$$

Therefore we have:

$$\begin{aligned}
\sin 2x &= 2.\sqrt{1-t}.\sqrt{t} \\
&= 2.\sqrt{t(1-t)} \\
&= \sqrt{4t(1-t)}
\end{aligned}$$

Solution T24:

$$\begin{aligned}
\frac{\sin x \cos x}{1+\cos^2 x - \sin^2 x} &= 0 \\
\therefore \frac{\sin x \cos x}{1-\sin^2 x + \cos^2 x} &= 0 \\
\therefore \frac{\sin x \cos x}{\cos^2 x + \cos^2 x} &= 0 \\
\therefore \frac{\sin x \cos x}{2\cos^2 x} &= 0 \\
\therefore \frac{\sin x}{2\cos x} &= 0 \\
\therefore \frac{1}{2}\frac{\sin x}{\cos x} &= 0 \\
\therefore \frac{1}{2}\tan x &= 0 \\
\therefore \tan x &= 0
\end{aligned}$$

Now we find the general solution for $\tan x$:

$$\begin{aligned}
x &= \tan^{-1}(0) + k.180^\circ \qquad k \in \mathbb{Z} \\
\therefore x &= 0^\circ + k.180^\circ
\end{aligned}$$

Now we use our general solution to find the values of x such that $x \in (-180^\circ; 180^\circ]$, by selecting different values for k:

For k=0:

$$\begin{aligned}
x &= 0^\circ + 0.180^\circ \\
\therefore x &= 0^\circ
\end{aligned}$$

$0^\circ \in (-180^\circ; 180^\circ]$. Therefore $x = 0^\circ$ is a solution.

For k=1:

$$\begin{aligned}
x &= 0^\circ + 1.180^\circ \\
\therefore x &= 180^\circ
\end{aligned}$$

$180^\circ \in (-180^\circ; 180^\circ]$. Therefore $x = 180^\circ$ is a solution.

For k=-1:

$$x = 0^\circ + (-1).180^\circ$$
$$\therefore x = -180^\circ$$

$-180^\circ \notin (-180^\circ; 180^\circ]$. Therefore $x = -180^\circ$ is not a solution.

We can see that for $k > 1$ or $k < -1$ there will not be a solution as the result will fall outside of the interval $(-180^\circ; 180^\circ]$.

Solution T25:

Looking at the graph, it is clear where $f(x)$ is greater than $g(x)$, between the two points of intersection. However, we must find the points of intersection of the two graphs so that we can define the interval precisely. To do this, we equate the two functions:

$$2\sin x = \cos 2x$$
$$\therefore 2\sin x = 1 - 2\sin^2 x$$
$$\therefore 2\sin^2 x + 2\sin x - 1 = 0$$

If we let $\sin x = k$, we can see that we have a quadratic expression:

$$2k^2 + 2k - 1 = 0$$

We have to use the formula to solve this quadratic:

$$k = \frac{-2 \pm \sqrt{2^2 - 4(2)(-1)}}{2(2)}$$

Therefore $k = 0.366$ or -1.366.
Therefore $\sin x = 0.366$. We reject -1.366 because $\sin x$ cannot be less than -1.

We find the general solution for $\sin x$:

$$x = \sin^{-1} 0.366 + n.360^\circ \qquad \text{OR} \qquad x = 180^\circ - \sin^{-1} 0.366 + n.360^\circ \qquad n \in \mathbb{Z}$$

Therefore:

$$x = 21.47^\circ + n.360^\circ \qquad \text{OR} \qquad x = 158.53 + n.360^\circ \qquad n \in \mathbb{Z}$$

We get the values for the two intersects when $n = 0$ for each part of the general solution. Therefore we have:

$$f(x) > g(x) \quad \text{for} \quad x \in (21.47; 158.53)$$

Note the use of round brackets, we do not include the intersection points because $f(x) = g(x)$ at these points and we were not asked to find the values of x for which $f(x) \geq g(x)$. If we were, then we would have used square brackets to include the intersection points.

Solution T26:

To find AC:

$$\begin{aligned}\sin 55^\circ &= \frac{50}{AC}\\ \therefore AC &= \frac{50}{\sin 55^\circ}\\ \therefore AC &= 61.04m\end{aligned}$$

To find AD:

$$\begin{aligned}\sin 48^\circ &= \frac{50}{AD}\\ \therefore AD &= \frac{50}{\sin 48^\circ}\\ \therefore AC &= 67.28m\end{aligned}$$

To find CD, we use the cosine rule:

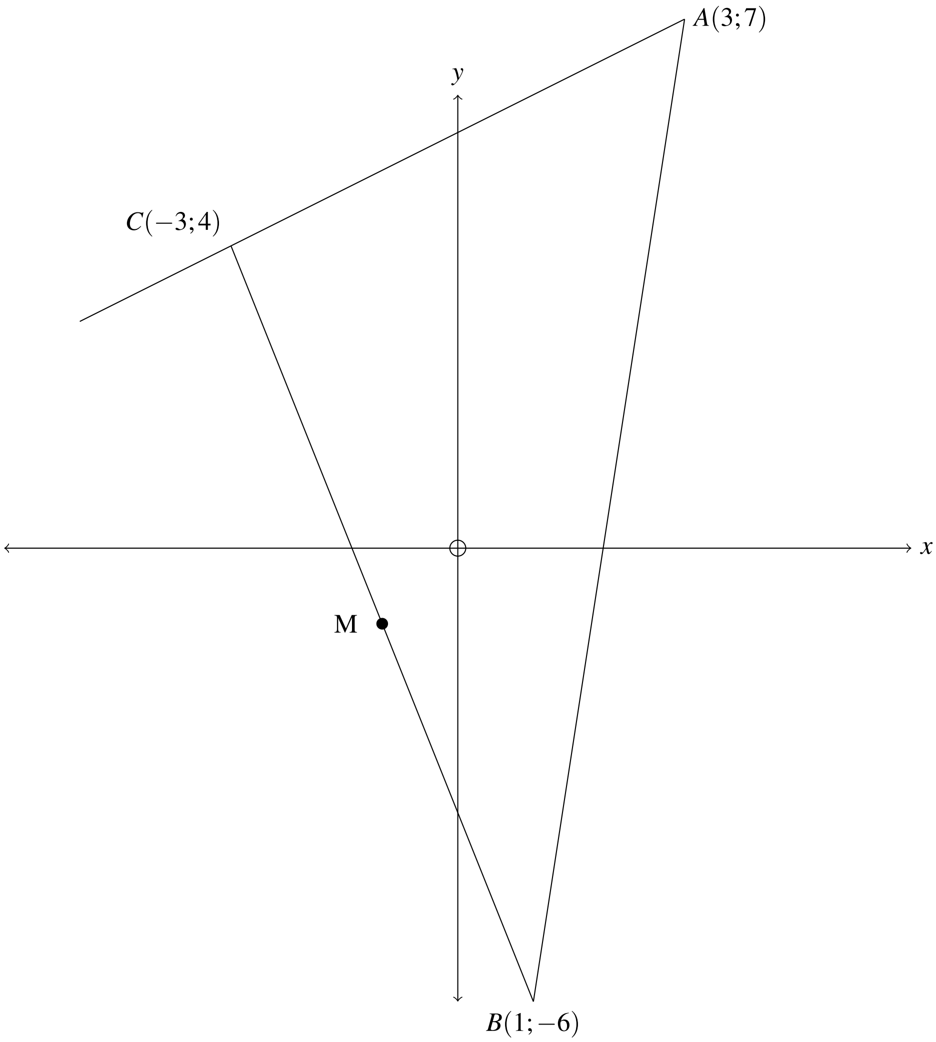
$$\begin{aligned}CD^2 &= AC^2 + AD^2 - 2AC.AD.\cos(71^\circ)\\ \therefore CD^2 &= 61.04^2 + 67.28^2 - 2.61.04.67.28.\cos(71^\circ)\\ \therefore CD^2 &= 5578.47\\ \therefore CD &= 74.69m\end{aligned}$$

Geometry Questions

Easy Geometry

Question G1:

Determine the coordinates of M, the midpoint of BC.

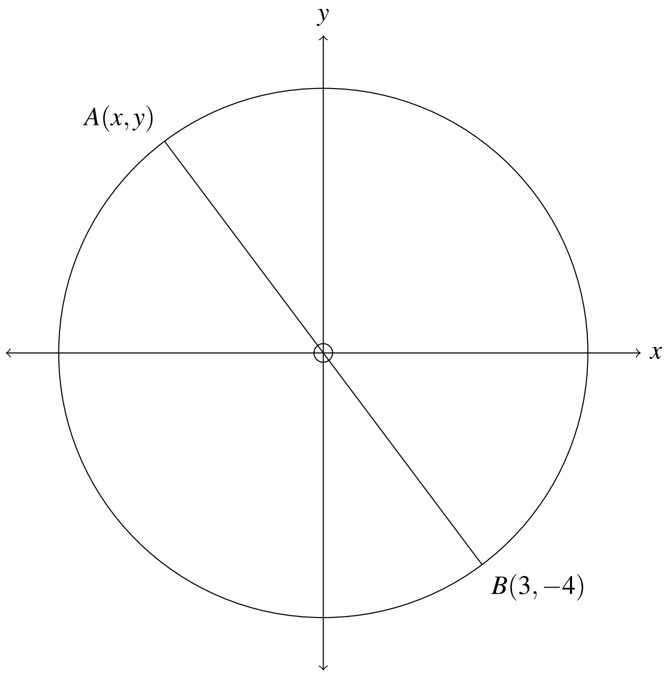


Hint

M is half way between the *x* and *y* vales of B and C.

Question G2:

Determine the coordinates of point A.

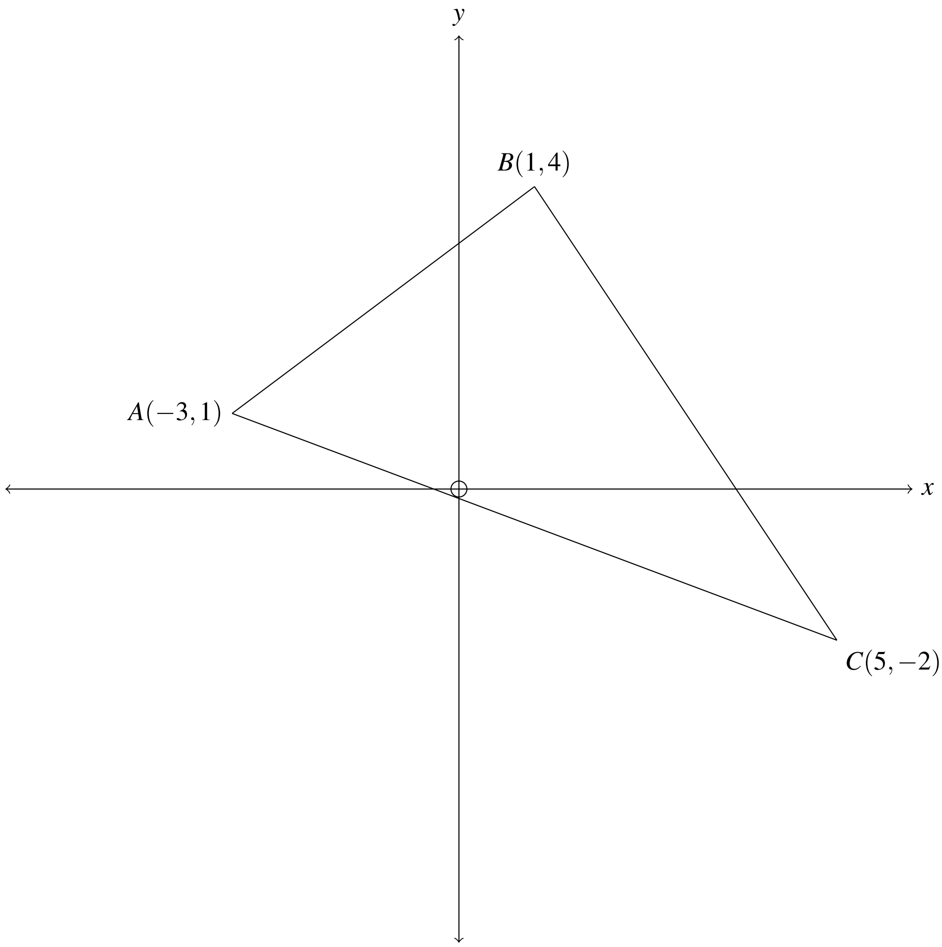


Hint

A is the $180°$ rotation transformation of B.

Question G3:

D is a point such that AD is parallel to to BC. Determine the equation of the line AD in the form $y = mx + c$.

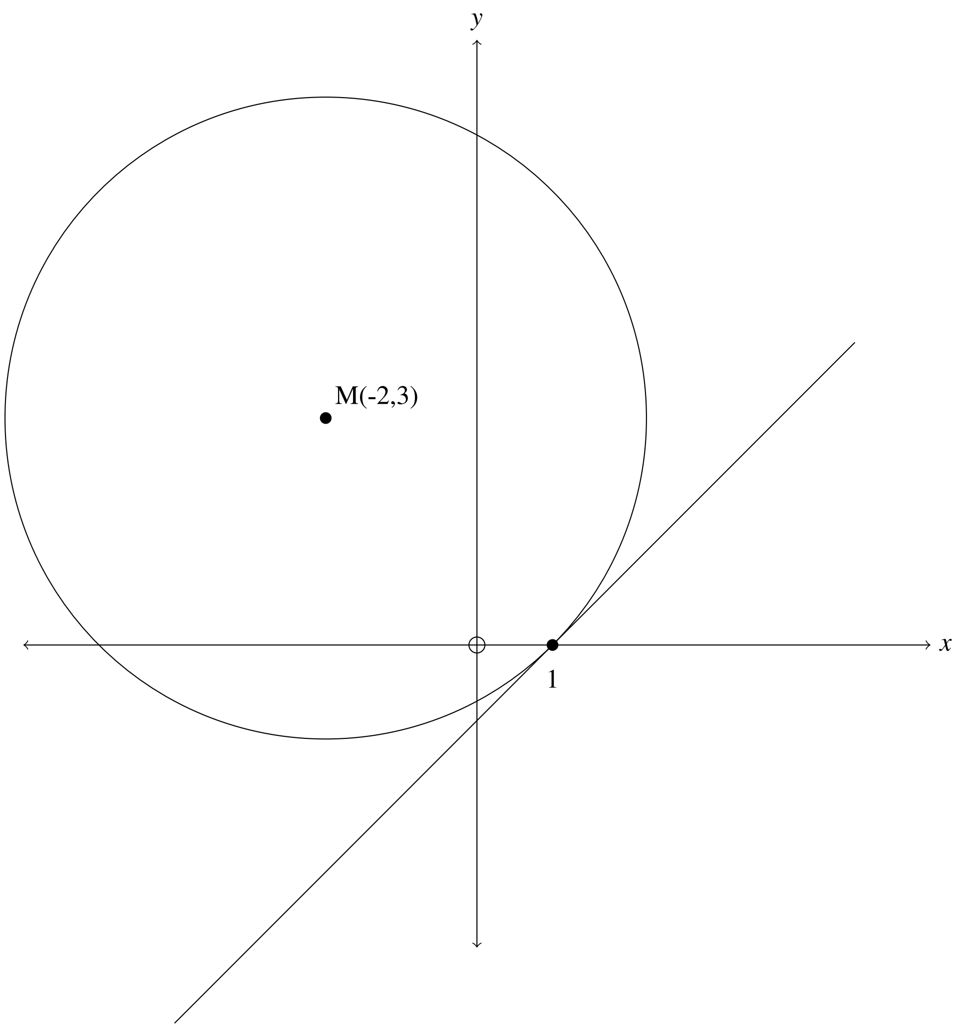


Hint

AD and BC have equal gradients.

Question G4:

Find the gradient of the tangent to the circle centred at M.

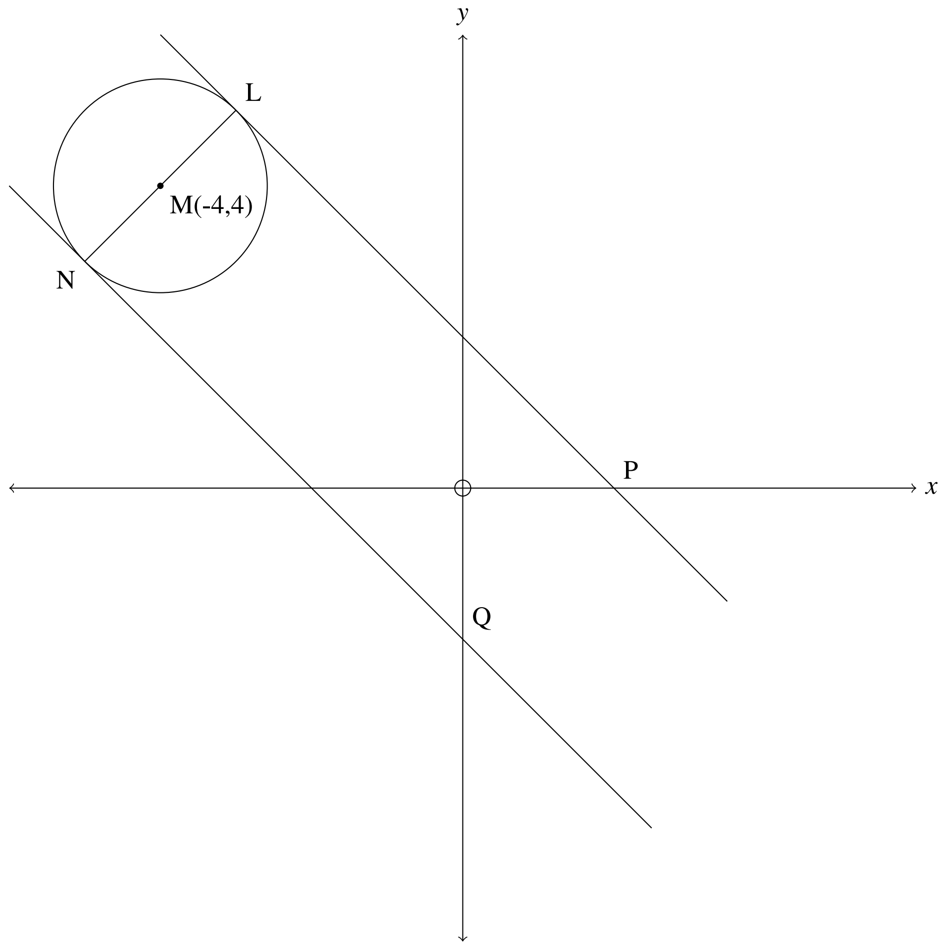


Hint

The line from M to the point of tangency is perpendicular to the tangent.

Question G5:

The line LP, with equation $y+x-2=0$, is a tangent at L to the circle with centre M(-4,4). Determine the equation of line NQ, in the form $y=mx+c$.

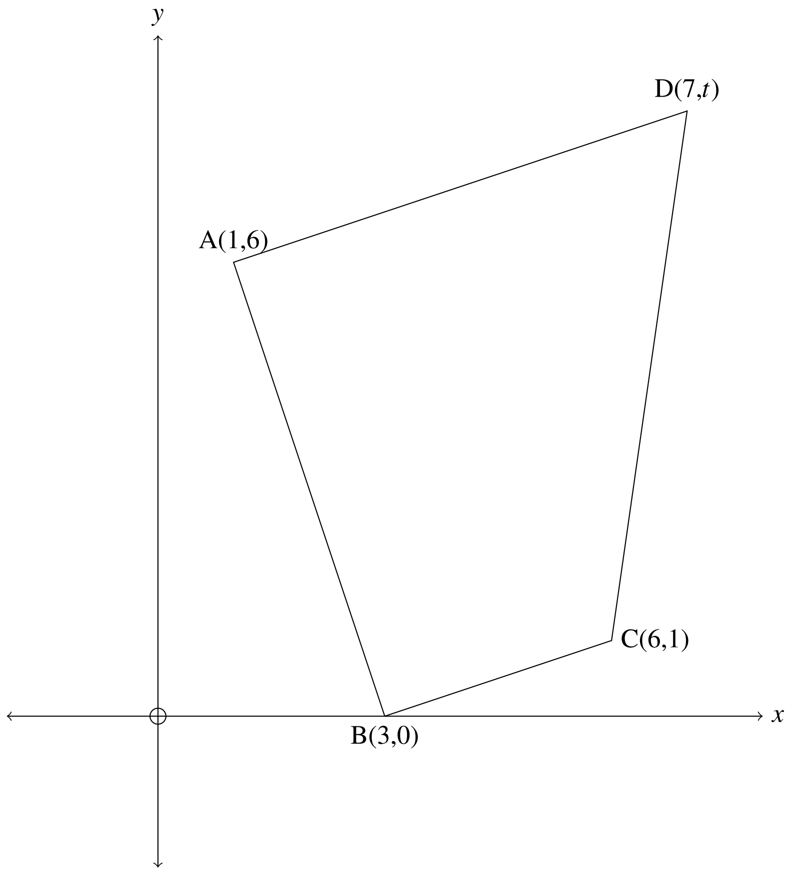


Hint

NQ is parallel to LP and they are symmetrical about the line running from the centre of the circle to the origin.

Question G6:

If AD is parallel to BC, determine the value of t for point D.

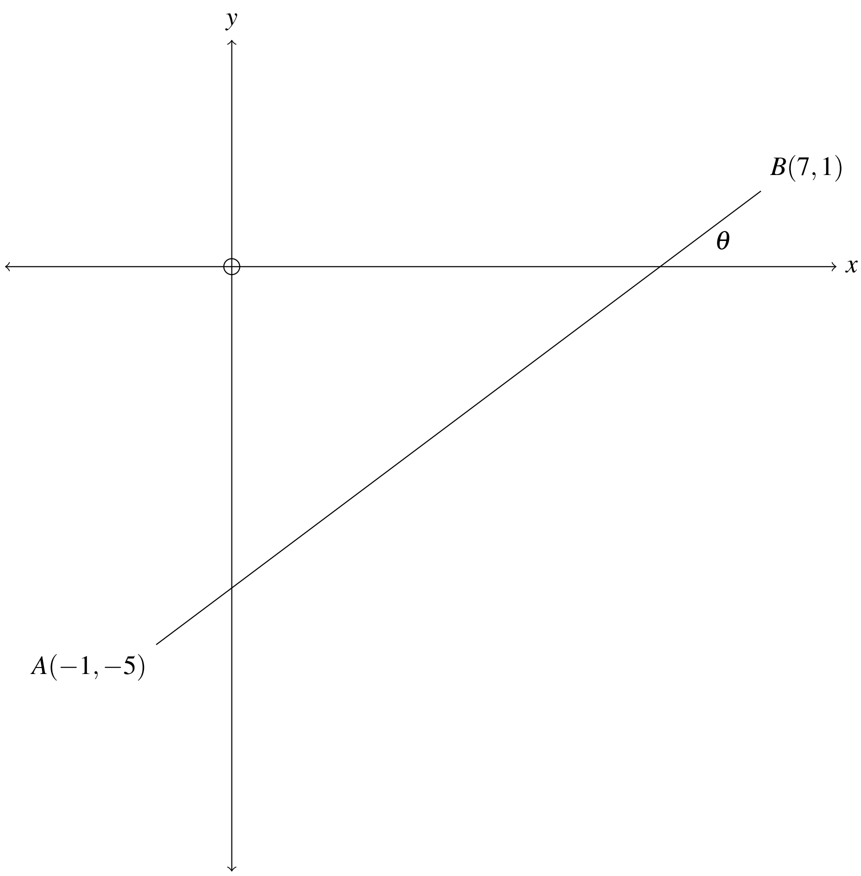


Hint

Start by finding the gradient of BC.

Question G7:

Find the angle, θ, that line AB makes with the x-axis.

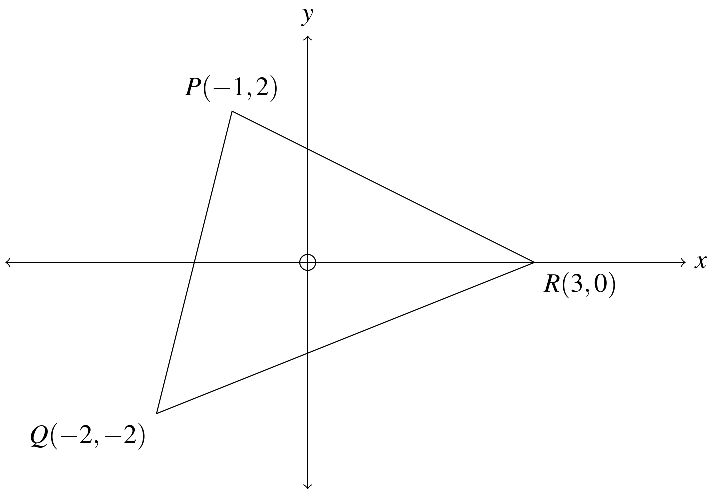


Hint

Start by finding the gradient of the line.

Question G8:

Determine the perimeter of $\triangle PQR$.

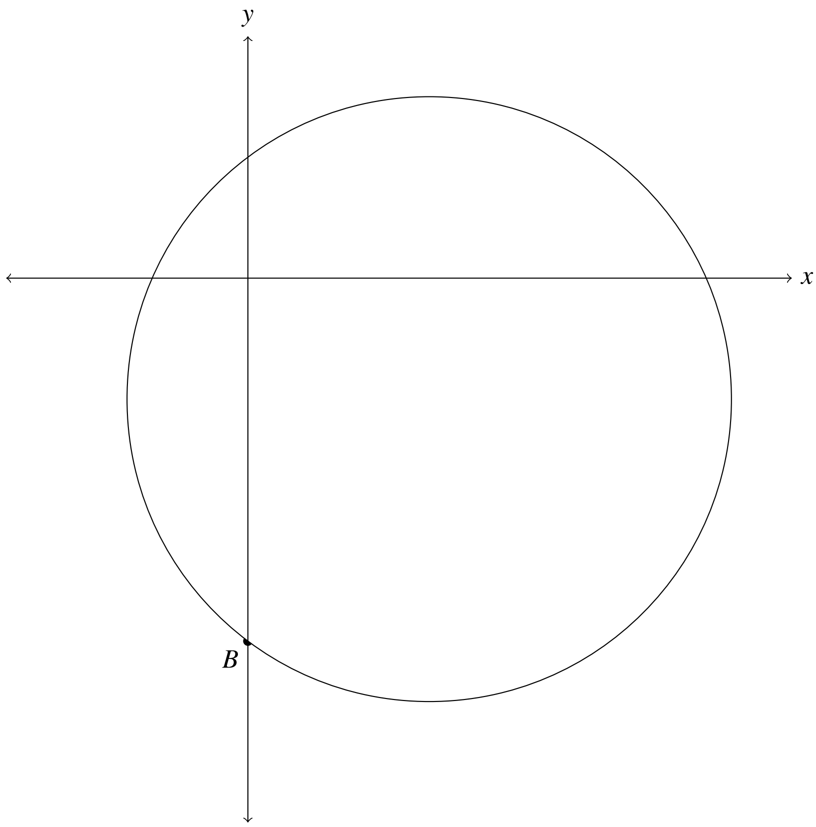


Hint

Use the distance formula: length $= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Question G9:

Determine the coordinates of point B. The equation of the circle is $(x-3)^2+(y+2)^2=25$.



Hint

You have the x-value of B.

Question G10:

Calculate the distance between the centres of of the two circles with equations $(x-3)^2+(y+2)^2=25$ and $(x-12)^2+(y-10)^2=100$.

Hint

The coordinates of the centres of the circles can be determined directly from their equations. For the first circle it's $(3,-2)$.

Question G11:

Determine the equation of the right side vertical tangent to the circle with equation $(x-12)^2+(y-10)^2=100$. This question does not require any calculation, drawing a rough labelled sketch may help.

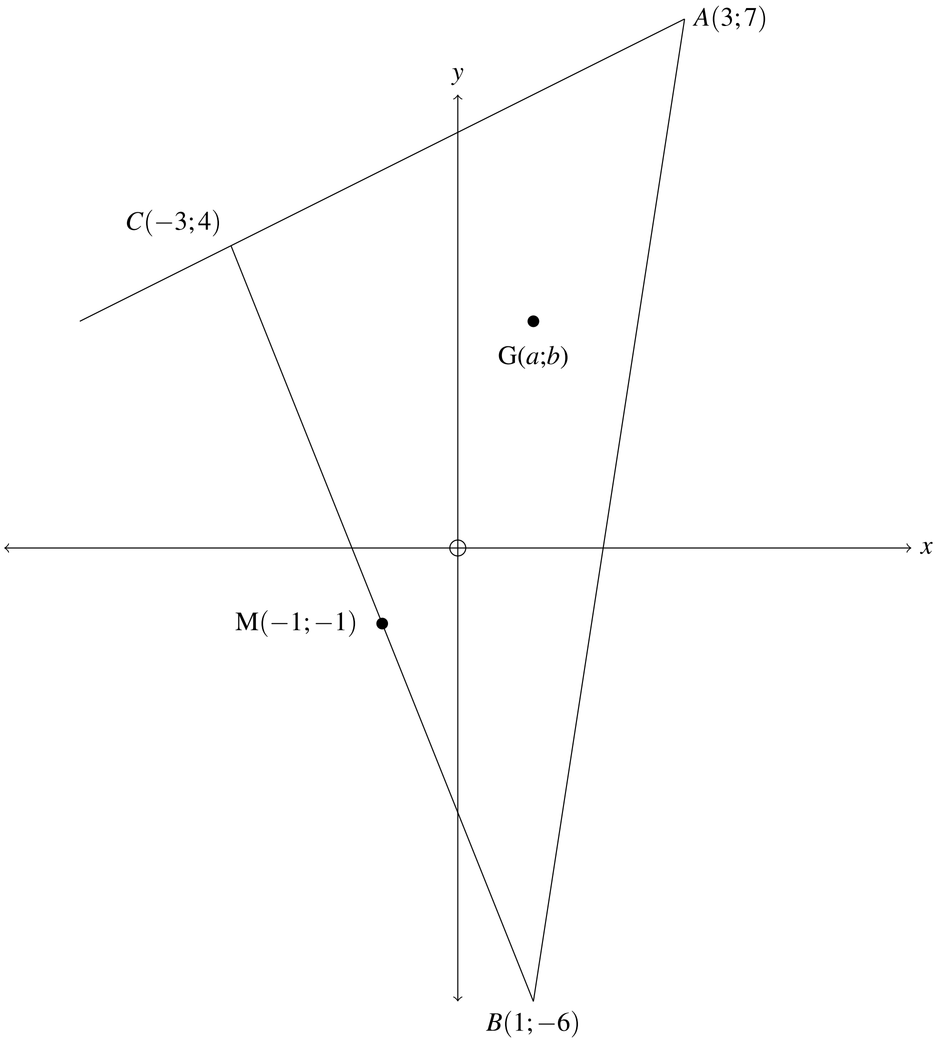
Hint

If the tangent is vertical then it is parallel to the y-axis, so it will have an equation of the form $x = c$, where c is a constant. To determine the value of x, think about how far the tangent is from the centre of the circle and the x-value of the coordinate of the centre of the circle. Then add those two values together.

Medium Geometry

Question G12:

If G(a;b) is a point such that A, G and M lie on the same straight line, find an expression that gives b in terms of a.

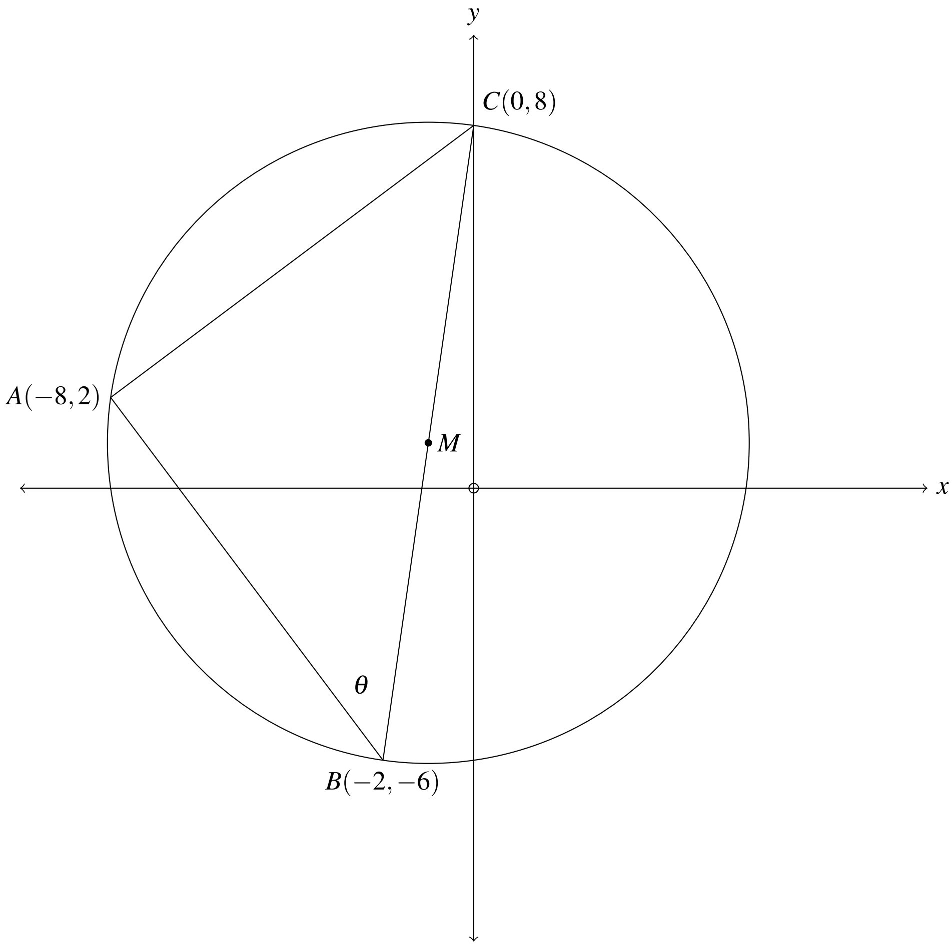


Hint

Find the equation of the straight line that runs through M and A.

Question G13:

A, B and C are the vertices of a triangle that lies on the circumfrence of a circle with centre M. Determine the size of angle θ.

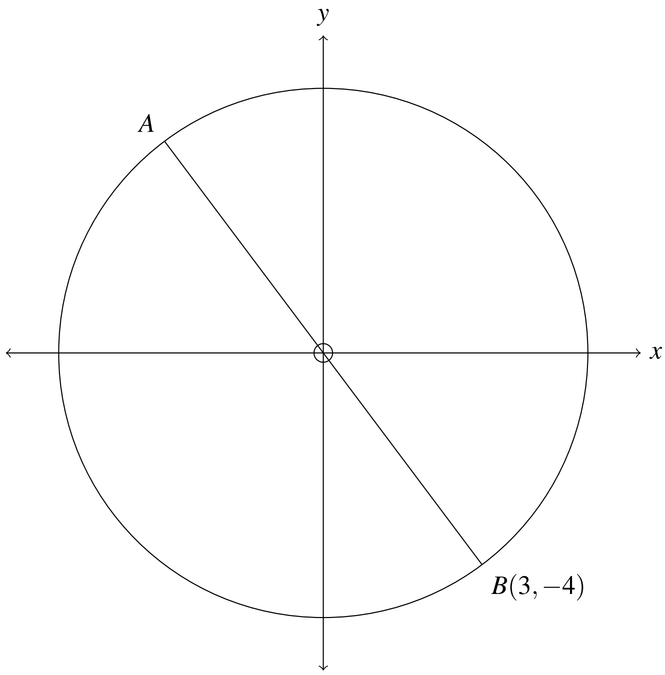


Hint

If BC is running through the centre of the circle and A is on the circumfrence, then $B\hat{A}C = 90°$. This is true no matter where A is on the circumfrence.

Question G14:

Determine the equation of the circle with centre B and radius AB, in the form $Ax^2 + Bx + Cy^2 + Dy + E = 0$.

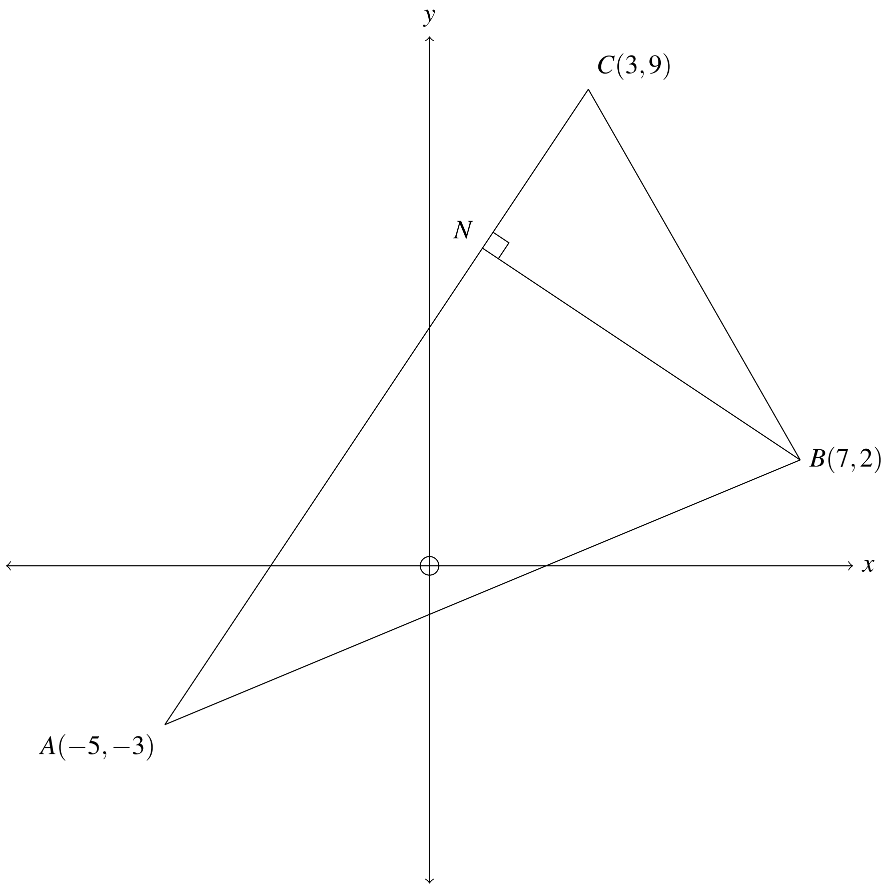


Hint

Start by determining the equation of the circle in the form $(x - x_0)^2 + (y - y_0)^2 = r^2$, where (x_0, y_0) is the centre of the circle. You have (x_0, y_0), now all you need is r.

Question G15:

Given $N = (1,6)$, calculate the area of triangle ABC.

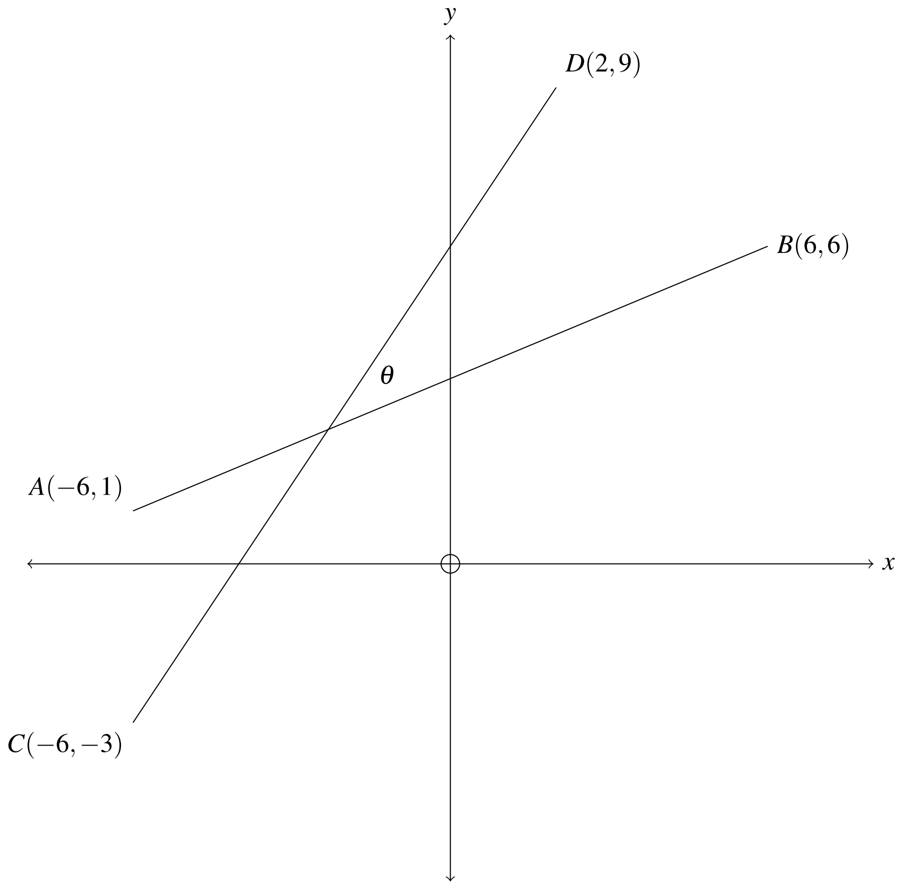


Hint

AC is the base of the triangle and BN is the height.

Question G16:

Calculate θ.



Hint

Find the angle that each line makes with the horizontal, by finding the gradient of each line.

Question G17:

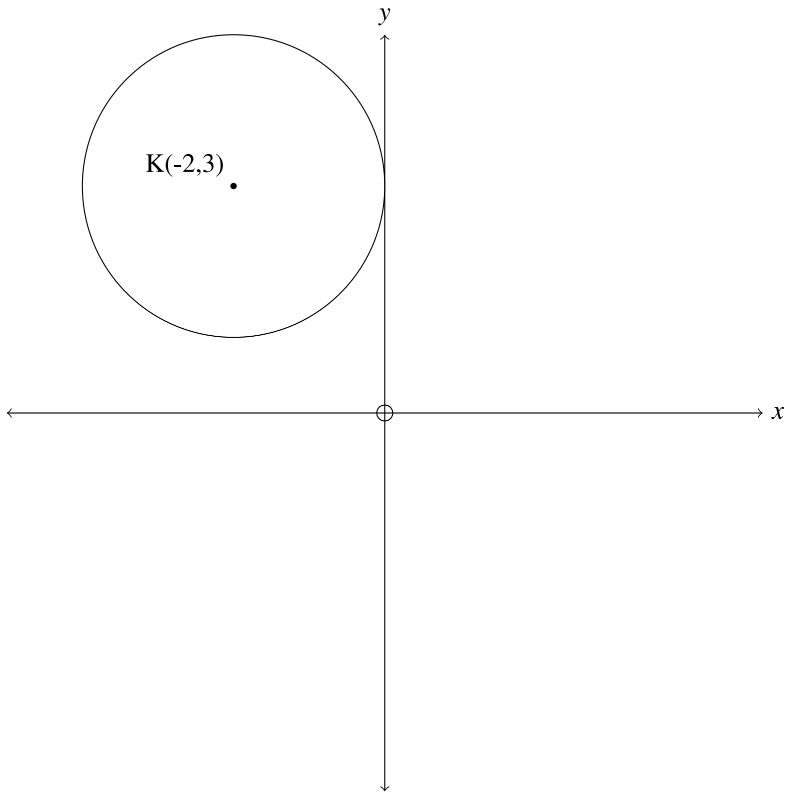
Determine the area of a triangle with sides of length 6cm, 9cm and 13cm.

Hint

Cosine rule and sine rule.

Question G18:

The circle centred at K(-2,3) touches the y-axis. If the length of its diameter is doubled and it is shifted 5 units to the right and 1 unit down, determine the equation of the new circle.



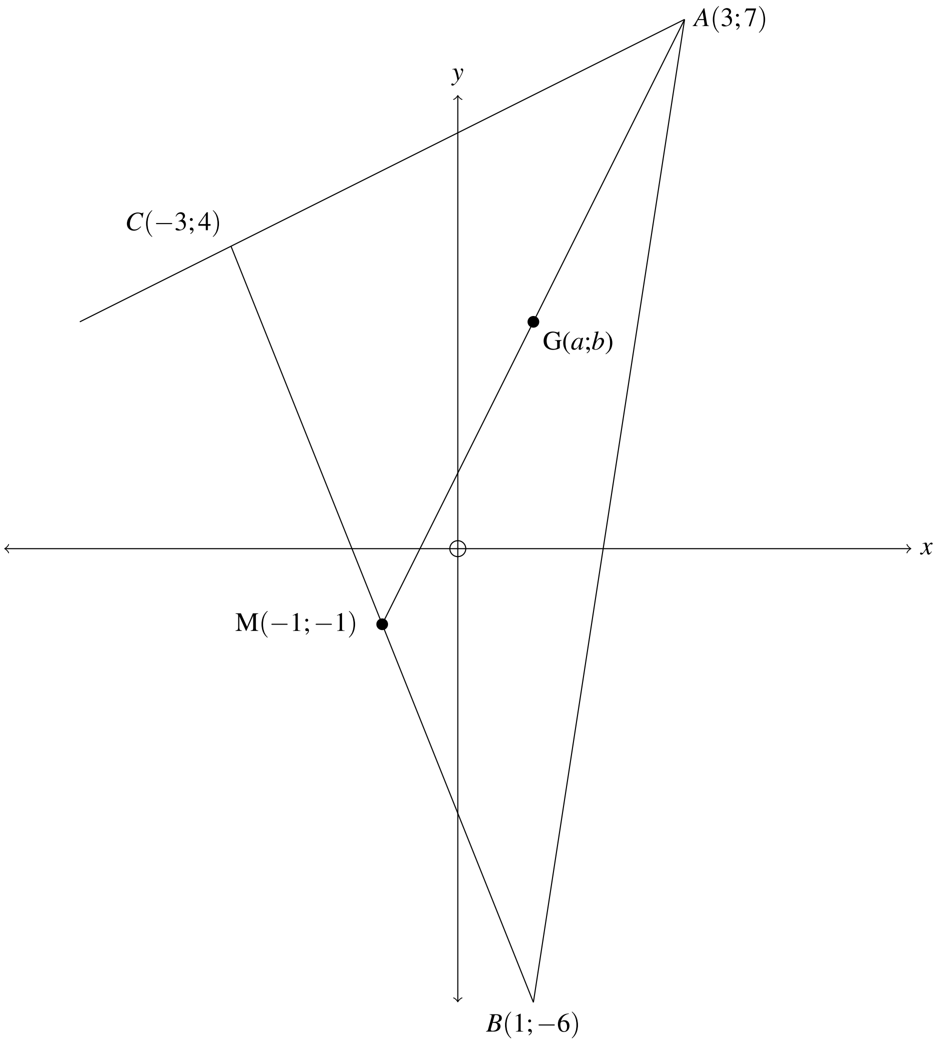
Hint

Subtract from x to shift to the right. Add to x to shift to the left. Subtract from y to shift upwards. Add to y to shift downwards.

Difficult Geometry

Question G19:

Given that the equation of the line running from A to M is $y = 2x + 1$ and G(a;b) is a point anywhere on the line, calculate the two possible values of b if GC = $\sqrt{17}$.

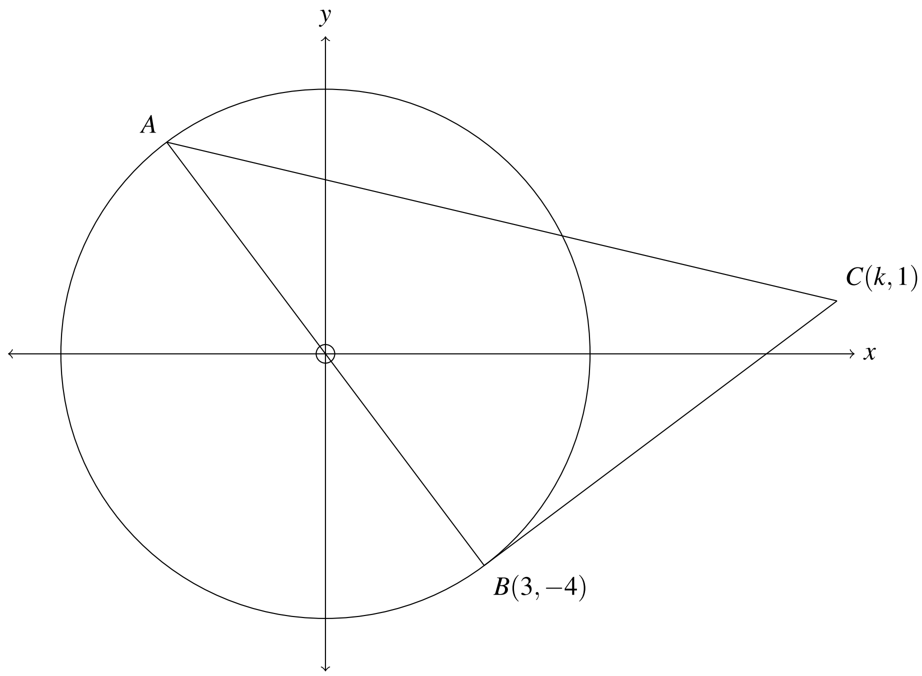


Hint

Express the distance between C and G in terms of a and b. Use the equation of the line between A and M to rewite this expression in terms of one variable.

Question G20:

If the line BC is a tangent to the circle, with centre $(0,0)$, at B, determine the value of k.

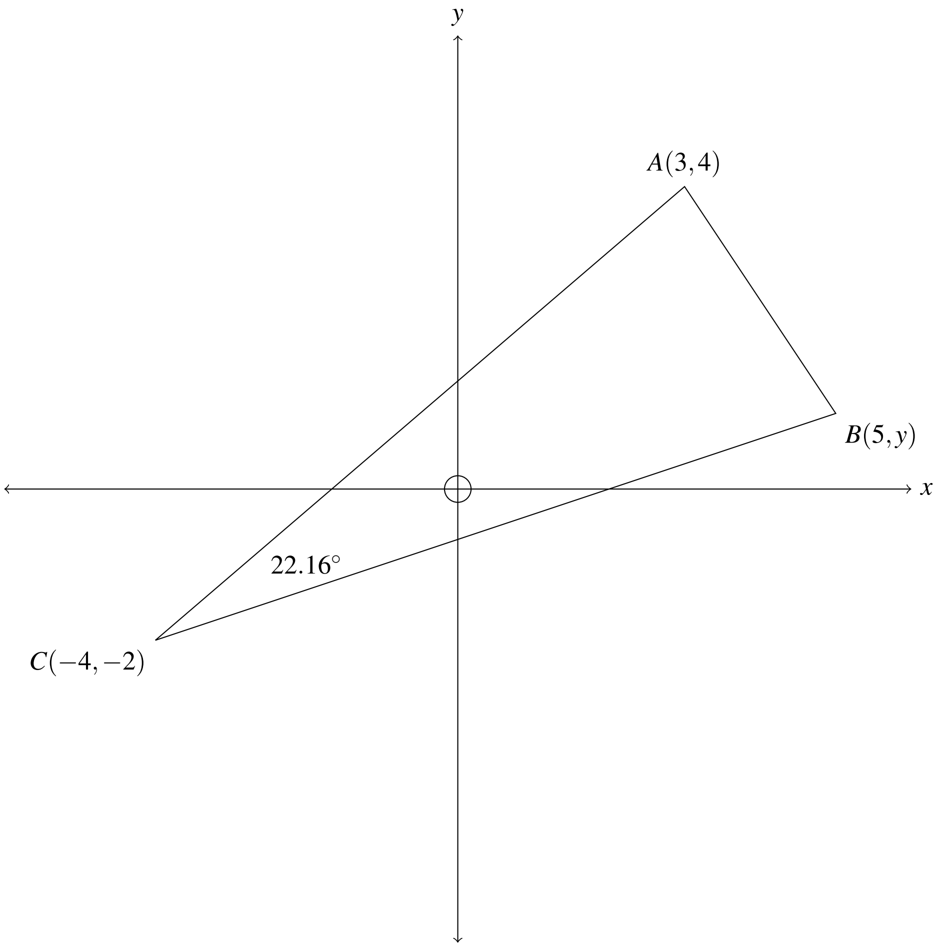


Hint

You need to do is find the equation of the line BC. You have a point, now all you need is a gradient. What is the angle between a tangent to a point on the circumfrence of a circle and the radius that extends to that same point?

Question G21:

Determine the value of the y coordinate of point B.

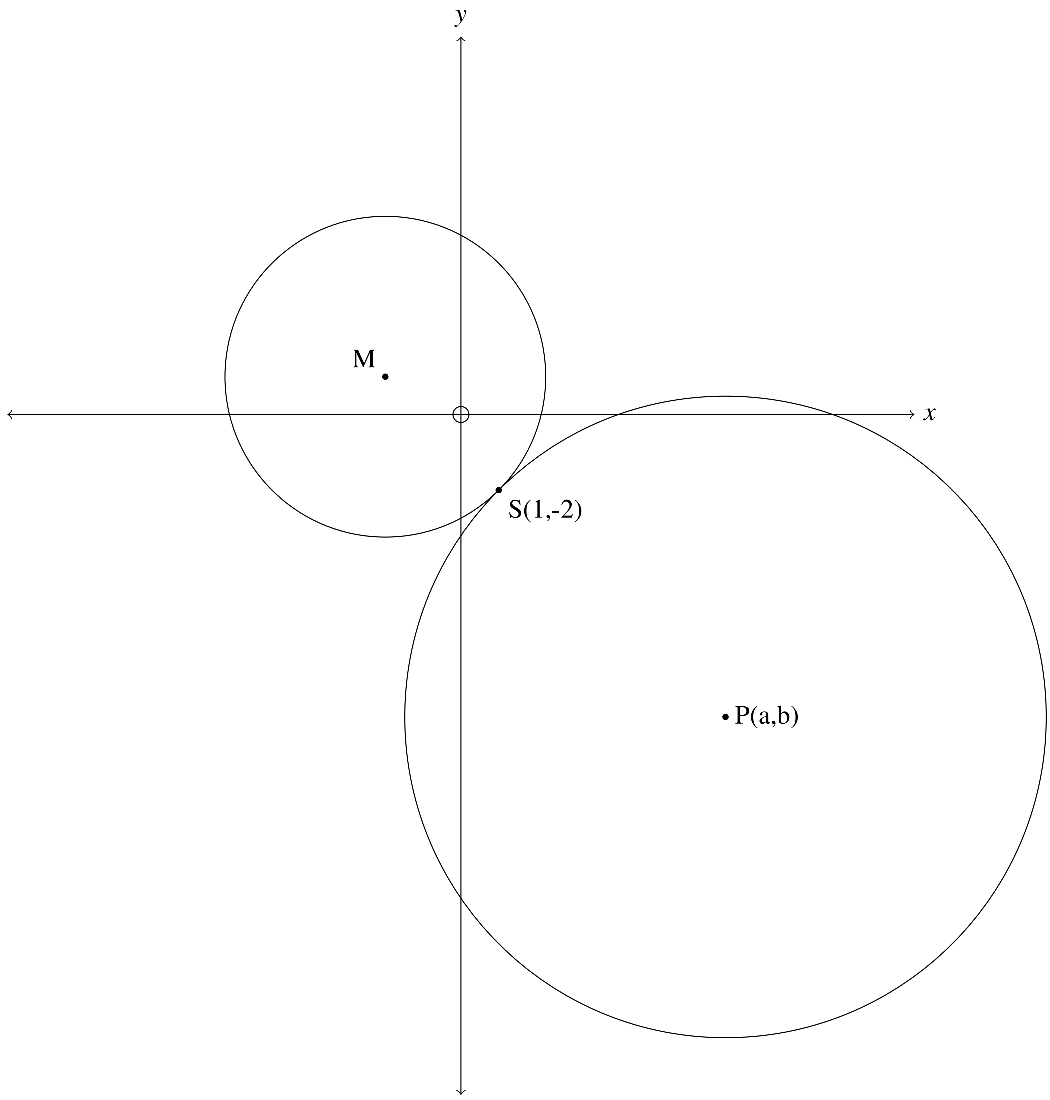


Hint

You need to find the equation of line CB. Start by determining the angle that line AC makes with the horizontal.

Question G22:

Find the point P if the equation of the circle centred at point M is $(x+2)^2+(y-1)^2=r$ and MS:MP = 1:3.

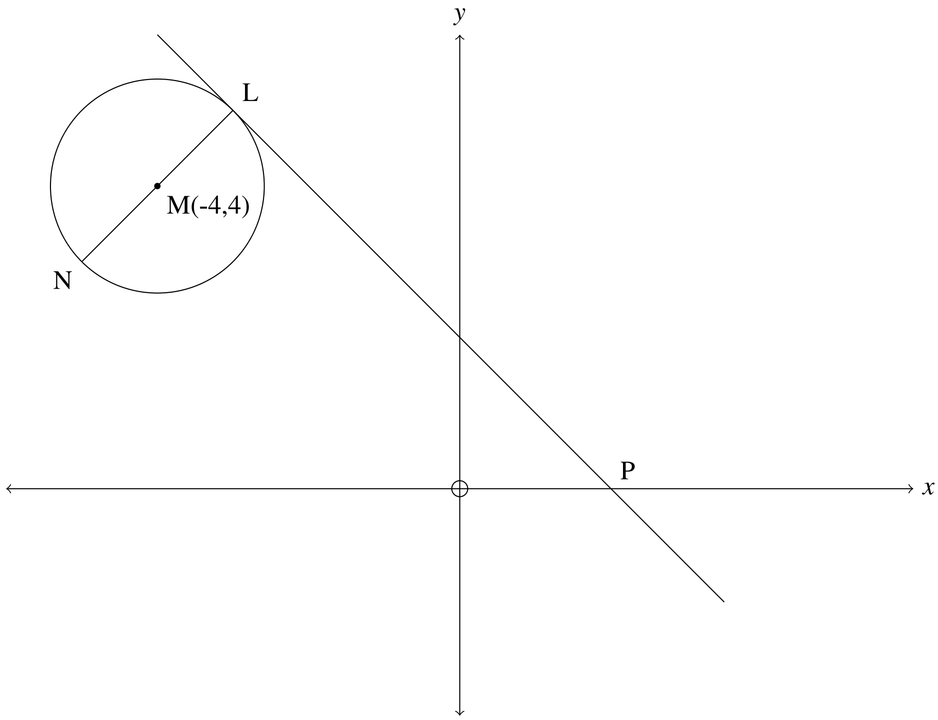


Hint

The ratio between the lengths of two line segments is equal to the ratio between the lengths of their corresponding horizontal or vertical components.

Question G23:

The line LP, with equation $y+x-2=0$, is a tangent at L to the circle with centre M(-4,4). LN is a diameter. Determine the equation of the circle.



Hint

You are looking for the radius of the circle. LN is perpendicular to LP. Start by finding the equation of LN by first finding its gradient.

Geometry Solutions

Solution G1:

$$M = \left(\frac{(-3)+(1)}{2}; \frac{(4)+(-6)}{2}\right)$$
$$= (-1,-1)$$

Solution G2:

A is the 180° rotation transformation of B. Therefore $A = (-3,4)$.

Solution G3:

AD and BC have equal gradients, so we first we find AD's gradient by finding gradient of BC:

$$m = \frac{y_1 - y_2}{x_1 - x_2}$$
$$\therefore m = \frac{4-(-2)}{1-5}$$
$$\therefore m = -1.5$$

So far we have $y = -1.5x + c$ as the equation of line AD. To find c we substitute point A into the equation:

$$y = -1.5x + c$$
$$\therefore 1 = -1.5(-3) + c$$
$$\therefore -3.5 = c$$
$$\therefore y = -1.5x - 3.5$$

Solution G4:

The angle between a radius and a tangent is 90°. Also, if two lines are perpendicular to one another then the gradient of one is equal to the negative inverse of the gradient of the other. So, if we find the gradient of radius then we can take it's negative inverse as the gradient of the tangent.

Gradient of the radius:

$$m_{rad} = \frac{y_1 - y_2}{x_1 - x_2}$$
$$\therefore m = \frac{3-0}{(-2)-1}$$
$$\therefore m = -1$$

Gradient of the tangent:

$$
\begin{aligned}
m_{tan} &= -\frac{1}{m_{rad}} \\
\therefore m &= -\frac{1}{-1} \\
\therefore m &= 1
\end{aligned}
$$

Solution G5:

NL is a diameter. Therefore, lines LP and NQ are both perpendicular to NL. Therefore, LP is parallel to NQ. Therefore LP and NQ have the same gradient, which is -1.

To find the y-intercept of NQ we notice that LP and NQ are both equidistant from the line MO (which runs from the circle centre to the origin). This is true because the centre of the circle is at (-4,4), which means that MO has a gradient of -1 and is therefore parallel to LP and NQ. Therefore, the distance of the y-intercepts of LP and NQ, from the origin, must be equal. Therefore, the intercept of NQ must be -2.

Putting this together we get the following equation for NQ:

$$y = -x - 2$$

Solution G6:

AD is parallel to BC. Therefore, the gradient of BC is equal to that of AD. If we have AC's gradient, we can easily find t.

Finding BC's gradient:

$$
\begin{aligned}
m &= \frac{y_1 - y_2}{x_1 - x_2} \\
\therefore m_{BC} &= \frac{1-0}{6-3} \\
\therefore m_{BC} &= \frac{1}{3} \\
\therefore m_{AD} &= \frac{1}{3}
\end{aligned}
$$

We can now use the gradient formula in reverse to find t:

$$
\begin{aligned}
m_{AD} &= \frac{1}{3} \\
\therefore \frac{t-6}{7-1} &= \frac{1}{3} \\
\therefore \frac{t-6}{6} &= \frac{1}{3} \\
\therefore t - 6 &= 2 \\
\therefore t &= 8
\end{aligned}
$$

Solution G7:

We have that $\tan\theta = m_{AB}$.

Here's the reason for this:

The gradient of a line is equal to $\dfrac{\Delta y}{\Delta x}$. $\tan\theta$ is equal to $\dfrac{\text{opposite}}{\text{adjacent}}$. We can see that, for an angle θ that a line makes with the x-axis, any triangle containing θ has opposite side equal to a change in y and the adjacent side equal to a change in x. Therefore $\tan\theta$ is equal to $\dfrac{\Delta y}{\Delta x}$. Therefore $\tan\theta$ is equla to the gradient

First we find AB's gradient:

$$\begin{aligned}
m &= \frac{y_1 - y_2}{x_1 - x_2} \\
\therefore m_{AB} &= \frac{7-(-1)}{1-(-5)} \\
\therefore m_{AB} &= \frac{8}{6} \\
\therefore m_{AB} &= \frac{4}{3}
\end{aligned}$$

Then we use this gradient to solve for θ.

$$\begin{aligned}
\tan\theta &= m_{AB} \\
\therefore \theta &= \tan^{-1}(m_{AB}) \\
\therefore \theta &= \tan^{-1}\left(\frac{4}{3}\right) \\
\therefore \theta &= 53.13^\circ
\end{aligned}$$

Solution G8:

Length PQ:

$$\begin{aligned}
&\sqrt{(x_1-x_2)^2+(y_1-y_2)^2} \\
=&\sqrt{((-1)-(-2))^2+(2-(-2))^2} \\
=&\sqrt{(1)^2+(4)^2} \\
=&\sqrt{17}
\end{aligned}$$

Length PR:

$$\begin{aligned}
&\sqrt{(x_1-x_2)^2+(y_1-y_2)^2} \\
=&\sqrt{((-1)-3)^2+(2-0)^2} \\
=&\sqrt{(-4)^2+(2)^2} \\
=&\sqrt{20}
\end{aligned}$$

Length QR:

$$
\begin{aligned}
&\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}\\
=&\sqrt{((-2)-3)^2+((-2)-0)^2}\\
=&\sqrt{(-5)^2+(-2)^2}\\
=&\sqrt{29}
\end{aligned}
$$

Perimeter:

$$
\begin{aligned}
\text{Perimeter} =& PQ+PR+QR\\
=&\sqrt{17}+\sqrt{20}+\sqrt{29}\\
=&13.98
\end{aligned}
$$

Solution G9:

Length PQ:

We have that the x-value of B is 0, because B sits on the y-axis. All we need to do is substitute 0 into the equation of the circle to find the y-coordinate.

$$
\begin{aligned}
((0)-3)^2+(y+2)^2 &= 25\\
\therefore 9+(y+2)^2 &= 25\\
\therefore (y+2)^2 &= 16\\
\therefore y+2 &= \pm 4\\
\therefore y = 2 \quad \text{OR} \quad y &= -6
\end{aligned}
$$

Clearly the y-value of B is -6. Therefore $B=(0,-6)$.

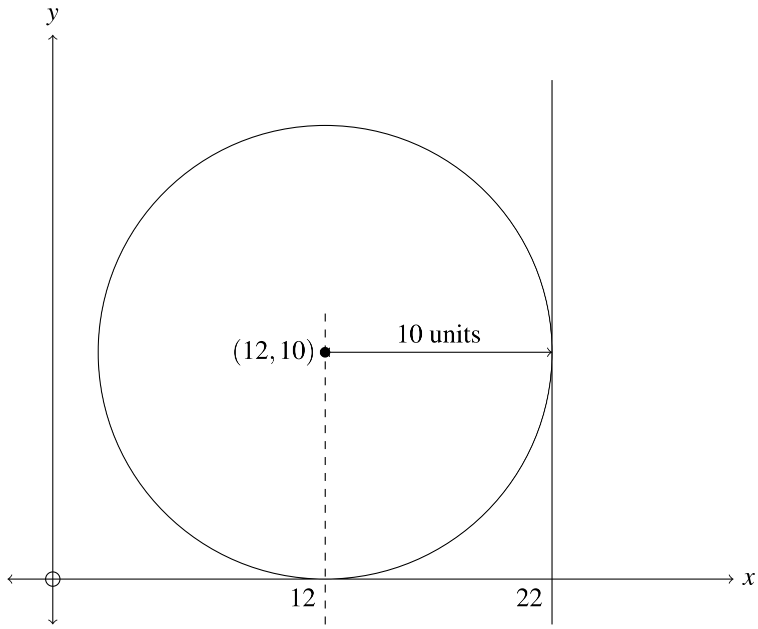
Solution G10:

The coordinates of the centres of the circles can be determined directly from their equations. For the first circle it's $(3,-2)$ and for the second circle it's $(12,10)$. Now we need only find the distance between these two ponts using the distance formula.

$$
\begin{aligned}
&\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}\\
=&\sqrt{(3-12)^2+((-2)-10)^2}\\
=&\sqrt{(9)^2+(-12)^2}\\
=&\sqrt{81+144}\\
=&\sqrt{225}\\
=&15
\end{aligned}
$$

Solution G11:

The circle has centre $(12,10)$ and radius 10 units. A right side vertical tangent to the circle will be 10 units to the right of its centre and perpendicular to the x-axis. Therefore it will have equation $x = 22$.



Solution G12:

We first have to find the equation of the straight line that runs through M and A.

$$\begin{aligned}
m =& \frac{7-(-1)}{3-(-1)} \\
=& 2 \\
\therefore y =& 2x + c \\
\therefore 7 =& 2(3) + c \\
\therefore c =& 1 \\
\therefore y =& 2x + 1
\end{aligned}$$

We know that $(a;b)$ is a point on this straight line so we can express b in terms of a by substituting $(a;b)$ into our equation, and we get:

$$b = 2a + 1$$

Solution G13:

We have that B$\hat{A}$C = 90°. Therefore we need only find AC and AB to then find θ using the inverse tan function, $\theta = \tan^{-1}\left(\frac{AC}{AB}\right)$.

$$
\begin{aligned}
AC &= \sqrt{((-8)-0)^2+(2-8)^2} \\
\therefore AC &= \sqrt{100} \\
\therefore AC &= 10
\end{aligned}
$$

$$
\begin{aligned}
AB &= \sqrt{((-8)-(-2))^2+(2-(-6))^2} \\
\therefore AB &= \sqrt{100} \\
\therefore AB &= 10
\end{aligned}
$$

$$
\begin{aligned}
\theta &= \tan^{-1}\left(\frac{AC}{AB}\right) \\
\therefore \theta &= \tan^{-1}\left(\frac{100}{100}\right) \\
\therefore \theta &= \tan^{-1}(1) \\
\therefore \theta &= 45^\circ
\end{aligned}
$$

Solution G14:

Start by determining the equation of the circle in the form $(x-x_0)^2+(y-y_0)^2 = r^2$, where (x_0, y_0) is the centre of the circle. We are given $(x_0, y_0) = (3, -4)$.

We are also given $r = AB$. To find AB we need only find the length of the radius, OB, of the given circle and multiply it by 2. Therefore:

$$
\begin{aligned}
OB &= \sqrt{(0-(3))^2+(0-(-4))^2} \\
\therefore OB &= 5 \\
\therefore AB &= 10 \\
\therefore r &= 10
\end{aligned}
$$

We now plug all our information into the standard equation and then expand it to get the required form:

$$
\begin{aligned}
(x-x_0)^2+(y-y_0)^2 &= r^2 \\
\therefore (x-3)^2+(y+4)^2 &= 10^2 \\
\therefore x^2-6x+9+y^2+8y+16 &= 100 \\
\therefore x^2-6x+y^2+8y-75 &= 0
\end{aligned}
$$

$$A = 1B = -6C = 1D = 8E = -75$$

Solution G15:

AC is the base of the triangle and BN is the height. We know that the area of the triangle is equal to $\frac{1}{2}$base$\times$height. Therefore we need only find the lengths of AC and BN to calculate the area.

$$\begin{aligned} AC &= \sqrt{(3-(-5))^2+(9-(-3))^2} \\ AC &= \sqrt{208} \end{aligned}$$

$$\begin{aligned} BN &= \sqrt{(7-1)^2+(2-6)^2} \\ BN &= \sqrt{52} \end{aligned}$$

$$\begin{aligned} Area \quad \triangle ABC &= \frac{1}{2}\sqrt{208}\sqrt{52} \\ &= 52 \end{aligned}$$

Solution G16:

The angle that line CD makes with the horizontal:

$$\begin{aligned} \tan\theta &= m_{CD} \\ \therefore \tan\theta &= \frac{9-(-3)}{2-(-6)} \\ \therefore \tan\theta &= \frac{12}{8} \\ \therefore \theta &= \tan^{-1}\left(\frac{3}{2}\right) \\ \therefore \theta &= 56.31^\circ \end{aligned}$$

The angle that line AB makes with the horizontal:

$$\begin{aligned} \tan\theta &= m_{AB} \\ \therefore \tan\theta &= \frac{6-(1)}{6-(-6)} \\ \therefore \tan\theta &= \frac{5}{12} \\ \therefore \theta &= \tan^{-1}\left(\frac{5}{12}\right) \\ \therefore \theta &= 22.62^\circ \end{aligned}$$

By subtracting line AB's angle from line CD's angle, we arrive at θ:

$$56.31^\circ - 22.62^\circ = 33.69^\circ$$

Solution G17:

You need to determine one of the angles in the triangle. For example, I have chosen the angle between the sides of length 6cm and 9cm (you may have chosen to determine either of the other two angles):

$$\begin{aligned}
c^2 &= a^2 + b^2 - 2ab\cos C \\
\therefore 13^2 &= 6^2 + 9^2 - 2.6.9.\cos C \\
\therefore C &= \cos^{-1}\left(\frac{13^2 - 6^2 - 9^2}{-2.6.9}\right) \\
\therefore C &= 118.78^\circ
\end{aligned}$$

Now we can use the sine rule to determine the area of the triangle:

$$\begin{aligned}
\text{area} \quad \triangle ABC &= \frac{1}{2}ab\sin C \\
\therefore \text{area} \quad \triangle ABC &= \frac{1}{2}.6.9\sin 118.78^\circ \\
&= 23.66cm^2
\end{aligned}$$

Solution G18:

First we determine the equation of the circle centred at K. The general equation for a circle is $(x-a)^2 + (y-b)^2 = r^2$, where a and b are the x and y coordinates of the centre point, respectively. We have the centre point, it's (-2,3).

To find the radius we use the fact that the circle touches the y-axis. This means that the y-axis is tangential to the circle. Therefore, the radius is equal to the the distance from the centre of the circle to the y-axis, which is 2 (which is the magnitude of the x coordinate of the centre of the circle).

Putting this together we get the following equation for the circle centred at K:

$$\begin{aligned}
(x-(-2))^2 + (y-(3))^2 &= 2^2 \\
\therefore (x+2)^2 + (y-3)^2 &= 4
\end{aligned}$$

Now we can make the changes. To shift the circle 5 units to the right, we subtract 5 from x. To shift the circle 1 unit downwards, we add 1 unit to y. To double the diameter is to double the radius, we multiply the radius by 2 BEFORE we square it. Putting this all together we get the folowing equation for the new circle:

$$\begin{aligned}
((x-5)+2)^2 + ((y+1)-3)^2 &= (2\times 2)^2 \\
\therefore (x-3)^2 + (y-2)^2 &= 16
\end{aligned}$$

Solution G19:

Distance between C and G:

$$\sqrt{17} = \sqrt{(a-(-3))^2 + (b-(4))^2}$$

$$
\begin{aligned}
\text{We have }\ y &= 2x+1 \\
\therefore b &= 2a+1 \\
\therefore 17 &= (a-(-3))^2+(2a+1-(4))^2 && \text{We square both sides to get rid of the square roots.} \\
\therefore 17 &= (a+3)^2+(2a-3)^2 \\
\therefore 17 &= a^2+6a+9+4a^2-12a+9 \\
\therefore 17 &= a^2+6a+9+4a^2-12a+9 \\
\therefore 0 &= 5a^2-6a+1 \\
\therefore 0 &= (5a-1)(a-1) \\
\therefore a &= \frac{1}{5} \quad or \quad 1 \\
\therefore b &= \frac{7}{5} \quad or \quad 3
\end{aligned}
$$

$$(1/5,7/5)or(1,3)$$

Solution G20:

To find k we first need to find the equation of the line BC. To do this, we need to find the gradient of BC. We are given that BC is a tangent to the circle at B and the circle has its centre at $(0,0)$, therefore OB is a radius from $(0,0)$ to B. We have a theorem which states that the angle, between a tangent to a point on the circumfrence of a circle and the radius that extends to that same point, is equal to $90°$. This means that OB is perpendicular to BC. Therefore, if we can find the gradient of OB then we can take its negative inverse as the gradient of BC.

$$
\begin{aligned}
m_{OB} &= \frac{(0-(-4))}{(0-3)} \\
\therefore m_{OB} &= -\frac{4}{3} \\
\therefore m_{BC} &= \frac{3}{4}
\end{aligned}
$$

We can set up an equation for BC:

$$
\begin{aligned}
y &= mx+c \\
\therefore y &= \frac{3}{4}x+c
\end{aligned}
$$

To find c, we substitute a known point, on BC, into the equation:

$$
\begin{aligned}
y &= \frac{3}{4}x+c \\
\therefore -4 &= \frac{3}{4}(3)+c \\
\therefore -\frac{25}{4} &= c \\
\therefore y &= \frac{3}{4}x-\frac{25}{4}
\end{aligned}
$$

To find k, we substitute $(k,1)$ into our equation:

$$\begin{aligned}
\therefore y &= \frac{3}{4}x - \frac{25}{4} \\
\therefore 1 &= \frac{3}{4}k - \frac{25}{4} \\
\therefore \frac{29}{3} &= k
\end{aligned}$$

Solution G21:

Determine the angle that line AC makes with the horizontal:

$$\begin{aligned}
\tan\theta &= m_{AC} \\
\therefore \tan\theta &= \frac{4-(-2)}{3-(-4)} \\
\therefore \tan\theta &= \frac{6}{7} \\
\therefore \theta &= \tan^{-1}\left(\frac{6}{7}\right) \\
\therefore \theta &= 40.60^\circ
\end{aligned}$$

Subtract 22.16° from 40.60° to get the angle that CB makes with the horizontal:

$$40.60^\circ - 22.16^\circ = 18.44^\circ$$

Using this angle, find the gradient of line CB:

$$\begin{aligned}
m_{CB} &= \tan 18.44 \\
\therefore m_{CB} &= 0.33
\end{aligned}$$

Use the gradient of line CB and the point C to determine the equation of line CB:

$$\begin{aligned}
y &= mx + c \\
\therefore y &= 0.33x + c \\
\therefore (-2) &= 0.33(-4) + c \\
\therefore c &= -0.67 \\
\therefore y &= 0.33x - 0.67
\end{aligned}$$

By substituting the x coordinate of B into the equation of line CB, we get the value of the y coordinate:

$$\begin{aligned}
y &= 0.33x - 0.67 \\
\therefore y &= 0.33(5) - 0.67 \\
\therefore y &= 1
\end{aligned}$$

Solution G22:

From the equation for the circle centred at M, we know that the coordinates of point M are (-2,1).

So, the horizontal component of MS is 3 units and its vertical component is 3 units. If MS:MP = 1:3 then the horizontal component of MP must be 9 units and it's vertical component must also be 9 units.

We know that MSP forms a straight line as M and P are the centres of circles that touch at S. Therefore, a will be 9 units from the x component of M and b will be 9 units from the y component of M. Therefore P = (7,-8).

Solution G23:

We know that LN is perpendicular to LP because LN is a diameter, LP is a tangent and L is the point of tangency. Therefore, the gradient of LN is the negative inverse of the gradient of LP. We are given the equation of LP and, putting this equation into standard form, we can see that its gradient is -1. Therefore, the gradient of LN is 1.

Now that we have a gradient (m=1) and a point (-4,4) for LN, we can determine it's equation:

$$\begin{aligned} y &= mx + c \\ \therefore (4) &= (1)(-4) + c \\ \therefore 8 &= c \\ \therefore y &= x + 8 \end{aligned}$$

With this equation we can find the point L, by finding the intersection of LP and LN:

$$\begin{aligned} y = x + 8 \qquad &\text{AND} \qquad y + x - 2 = 0 \\ \therefore (x+8) + x - 2 = 0 & \\ \therefore 2x + 6 = 0 & \\ \therefore x = -3 & \\ \therefore y = 5 & \\ \therefore L = (-3, 5) & \end{aligned}$$

Now we can find the radius, r, of the circle by finding the distance between L and M:

$$\begin{aligned} r &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \\ \therefore r &= \sqrt{(-3 - (-4))^2 + (5 - 4)^2} \\ \therefore r &= \sqrt{(1)^2 + (1)^2} \\ \therefore r &= \sqrt{2} \end{aligned}$$

We now have the centre and the radius of the circle. Given the standard form of the equation of a circle, $(x-a)^2 + (y-b)^2 = r^2$, we can determine the equation of the circle as follows:

$$\begin{aligned} (x - (-4))^2 + (y - (4))^2 &= (\sqrt{2})^2 \\ \therefore (x+4)^2 + (y-4)^2 &= 2 \end{aligned}$$

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